The equivariant slice spectral sequence

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Lecture 2

$\Sigma^{\sigma} H_{C_2} \underline{\mathbb{Z}}$

$\Sigma^{\lambda} H_{C_4} \underline{\mathbb{Z}}$

- Bredon homology
- The slice filtration, general G
- ~~Examples: $\Sigma^{V} H_G \underline{\mathbb{Z}}$~~

Lecture 3

- Examples: $\Sigma^V H_G \underline{\mathbb{Z}}$
- The $RO(G)$-grading
- ~~Duality~~

Examples of slice towers $\Sigma^7 H_{C_2}\underline{\mathbb{Z}}$

Homework $\Sigma^n H_{C_2}\underline{\mathbb{Z}}$ n-slice for $0 \le n \le 6$

Ex $\Sigma^7 H_{C_2}\underline{\mathbb{Z}}$ Is this a 7-slice?

$$\Sigma^7 H_{C_2}\underline{\mathbb{Z}} \overset{?}{\simeq} \Sigma^{3\rho+1} H_{C_2}\underline{M}, \text{ some } \underline{M} \text{ w/ injective } R$$

$\downarrow \Sigma^{-3\rho}$

$$\Sigma^{7-3\rho} H_{C_2}\underline{\mathbb{Z}} \simeq \Sigma^1 H_{C_2} Q\underline{\mathbb{Z}}^{\sigma}$$ Is this a 1-slice?

NO

$$\underline{g} \hookrightarrow Q\underline{\mathbb{Z}}^{\sigma} \twoheadrightarrow \underline{\mathbb{Z}}^{\sigma}$$

$$(2)\ \Sigma^1 H\underline{g} \to \Sigma^1 HQ\underline{\mathbb{Z}}^{\sigma} \to \Sigma^1 H\underline{\mathbb{Z}}^{\sigma}\ (1)$$

$$(8)\ \Sigma^{1+3\rho} H\underline{g} \to \Sigma^7 H\underline{\mathbb{Z}} \to \Sigma^{1+3\rho} H\underline{\mathbb{Z}}^{\sigma}\ (7)$$

$$\Sigma^{1+3\rho} H\underline{g} = \Sigma^4 H\underline{g}$$

8 $\Sigma^4 Hg \to \Sigma^7 H\underline{\mathbb{Z}}$

$$\downarrow$$

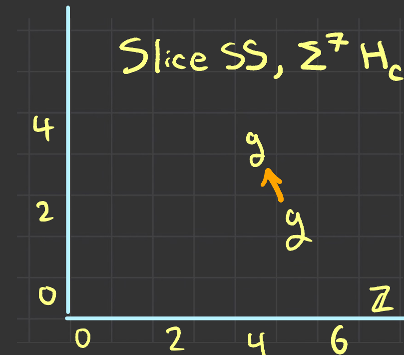
7 $\Sigma^{3+2\sigma} H\underline{\mathbb{Z}}^\sigma$

Examples of slice towers

8 $$\Sigma^{1+3\sigma} Hg \to \Sigma^7 H\underline{\mathbb{Z}} \to \Sigma^{1+3\sigma} H\underline{\mathbb{Z}}^\sigma$$ 7

$$\Sigma^{1+3\sigma} Hg = \Sigma^4 Hg \qquad \Sigma^{1+3\sigma} H\underline{\mathbb{Z}}^\sigma = \Sigma^{3+2\sigma} H\underline{\mathbb{Z}}$$

Slice SS, $\Sigma^7 H_{C_2}\underline{\mathbb{Z}}$



Ex $\Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}}$ 2-slice?

$$\downarrow \Sigma^{-\sigma}$$

$\Sigma^{\sigma-1} H_{C_2}\underline{\mathbb{Z}}$ 0-slice?

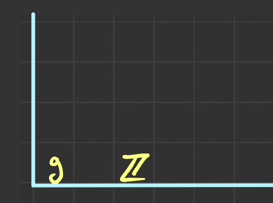
0 $$H_{C_2}\underline{\mathbb{Z}}^\sigma \to \Sigma^{\sigma-1} H_{C_2}\underline{\mathbb{Z}} \to \Sigma^{-1} H_{C_2} g$$ -2

$$\downarrow \Sigma^{\sigma}$$

$$\Sigma^{\sigma} H_{C_2}\underline{\mathbb{Z}}^\sigma \to \Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}} \to \Sigma^{\sigma} H_{C_2} g$$

2 $\Sigma^{\sigma} H_{C_2}\underline{\mathbb{Z}}^\sigma = \Sigma^2 H_{C_2}\underline{\mathbb{Z}}$ 0 $\Sigma^{\sigma} H_{C_2} g = H_{C_2} g$

Slice filt. for $\Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}}$ = Post. filt. for $\Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}}$



8 $\Sigma^4 H\underline{g} \to \Sigma^7 H\underline{\mathbb{Z}}$

$\downarrow$

7 $\Sigma^{3+2\sigma} H\underline{\mathbb{Z}}^{\sigma}$

2 $\Sigma^2 H_{C_2}\underline{\mathbb{Z}} \to \Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}}$

$\downarrow$

0 $H_{C_2}\underline{g}$

Examples of slice towers $\Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}}$

Ex $\Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}}$

- know have 2-slice, $2 = |\lambda|$ (detected on $\downarrow^{C_4}_{e}$)
- $\downarrow^{C_4}_{C_2} \Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Sigma^{2\sigma} H_{C_2}\underline{\mathbb{Z}}$ has 0,2 slices
- $P^0_0 \Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}} = H_{C_4}\underline{\pi}_0 \Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}} = H_{C_4}\underline{B}(2,0)$
- we found Postnikov filtration

$$P_1 = \Sigma^2 H_{C_4}\underline{\mathbb{Z}} \to \Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}} \to H_{C_4}\underline{B}(2,0) = P^0_0$$

Claim $\Sigma^2 H_{C_4}\underline{\mathbb{Z}}$ is a 2-slice.

$\Sigma^\lambda H_{C_4}\underline{\mathbb{Z}}$

$$\begin{array}{ccc} P_1 = \Sigma^2 H_{C_4}\underline{\mathbb{Z}} & \to & \Sigma^\lambda H_{C_4}\underline{\mathbb{Z}} \\ & & \downarrow \\ & & H_{C_4}\underline{B}(2,0) = P_0^0 \end{array}$$

Examples of slice towers $\Sigma^\lambda H_{C_4}\underline{\mathbb{Z}}$

Claim $\Sigma^2 H_{C_4}\underline{\mathbb{Z}}$ is a 2-slice.

- $\Sigma^2 H_{C_4}\underline{\mathbb{Z}}$ 2-connective $\Rightarrow$ slice 2-connective $(2 \geq 0)$
- $\Sigma^2 H_{C_4}\underline{\mathbb{Z}} \leq 2$

$\downarrow^{C_4}_{C_2} \Sigma^2 H_{C_4}\underline{\mathbb{Z}} = \Sigma^2 H_{C_2}\underline{\mathbb{Z}}$ a 2-slice.

Need $[S^{k\rho}, \Sigma^2 H_{C_4}\underline{\mathbb{Z}}]^{C_4} = 0$, $4k > 2$.

$$[S^\rho, \Sigma^2 H_{C_4}\underline{\mathbb{Z}}]^{C_4} = \pi_{-2}\left(\Sigma^{-\rho} H_{C_4}\underline{\mathbb{Z}}\right) = 0$$

So The Postnikov filtration for $\Sigma^\lambda H_{C_4}\underline{\mathbb{Z}}$ is the slice filtration.

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$\Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}}$

$P_2^2 = \Sigma^2 H_{C_4}\underline{\mathbb{Z}} \to \Sigma^{\lambda} H_{C_4}\underline{\mathbb{Z}}$

$\downarrow$

$H_{C_4}\underline{B}(2,0) = P_0^0$

The (regular) slice filtration

$|H| = |H|$

Hill-Yarnall $X \in Sp^G$

Then $X \geq n \iff \Phi^H X$ is $\frac{n}{|H|}$ - connective $\forall\, H \leq G$

vanishes below this

Ex

$\boxed{G = C_2}$

- $X \geq 1 \iff \downarrow_e^{C_2} X$ 1-connective & $\Phi^{C_2} X$ $1/2$-connective = 1-connective
- $X \geq 2 \iff \downarrow_e^{C_2} X$ 2-conn, & $\Phi^{C_2} X$ 1-conn

$X \geq 2 \iff$

$\downarrow_e^{C_2} X$ 2-conn,

& $\Phi^{C_2} X$ 1-conn

The (regular) slice filtration

Ex $\Sigma^1 H_{C_2}\underline{g}$: $\downarrow_e^{C_2} \Sigma^1 H_{C_2}\underline{g} \simeq *$ n-conn. $\forall n$

$\Phi^{C_2} \Sigma^1 H_{C_2}\underline{g} \simeq \Sigma^1 \Phi^{C_2} H_{C_2}\underline{g} \simeq \Sigma^1 H\mathbb{F}_2$ 1-conn

$\Rightarrow \Sigma^1 H_{C_2}\underline{g} \geq 2.$

Ex $\Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}}$: $\downarrow_e^{C_4} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Sigma^{-3} H\underline{\mathbb{Z}} \geq -3$

$\Phi^{C_2} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Sigma^{-1} \Phi^{C_2} H_{C_2}\underline{\mathbb{Z}} \geq -1$

$\Phi^{C_4} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Phi^{C_4} H_{C_4}\underline{\mathbb{Z}} \geq 0$

$\Rightarrow \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \geq -3$

$\downarrow_e^{C_2} \Sigma^1 H_{C_2}\underline{g} \simeq *,$

$\Phi^{C_2}\Sigma^1 H_{C_2}\underline{g} \simeq \Sigma^1 H\mathbb{F}_2$

$\Rightarrow \Sigma^1 H_{C_2}\underline{g} \geq 2$

$\downarrow_e^{C_4} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Sigma^{-3} H\underline{\mathbb{Z}} \geq -3$

$\Phi^{C_2} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Sigma^{-1}\Phi^{C_2} H_{C_2}\underline{\mathbb{Z}} \geq -1$

$\Phi^{C_4} \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \simeq \Phi^{C_4} H_{C_4}\underline{\mathbb{Z}} \geq 0$

$\Rightarrow \Sigma^{-\sigma-\lambda} H_{C_4}\underline{\mathbb{Z}} \geq -3$

RO(G)-grading

$RO(G)$ = real representation ring of G

$= \mathbb{Z}\{$iso. classes of irreps of $G/\mathbb{R}\}$ (choices of representatives)

Ex
- $RO(C_2) = \mathbb{Z}\{1, \sigma\}$
- $RO(C_3) = \mathbb{Z}\{1, \lambda\}$
- $RO(C_4) = \mathbb{Z}\{1, \sigma, \lambda\}$
- $RO(C_2 \times C_2) = \mathbb{Z}\{1, p_1^*\sigma, m^*\sigma, p_2^*\sigma\}$

RO(G)-grading $V, W \in Rep(G)$ $\quad S^{V-W} = S^V \wedge S^{-W}$

$$\pi_{V-W}(X) = [S^{V-W}, X]^G, \quad \pi^H_{V-W}(X) = [\downarrow^G_H S^{V-W}, \downarrow^G_H X]^H$$

$\rightsquigarrow$ RO(G)-graded Mackey functor $\underline{\pi}_\star X$

Ex $\underline{\pi}_{-\sigma} H_{C_2}\underline{\mathbb{Z}} \cong \underline{\pi}_0 \Sigma^\sigma H_{C_2}\underline{\mathbb{Z}} \cong \underline{g}$

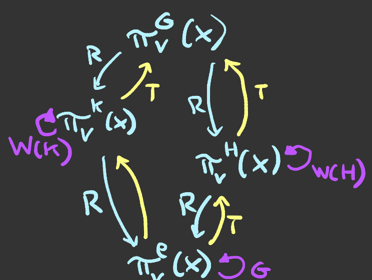
$RO(G) = \mathbb{Z}\{\text{iso. classes of irreps of } G\}$

$RO(C_2) = \mathbb{Z}\{1, \sigma\}$

$RO(C_4) = \mathbb{Z}\{1, \sigma, \lambda\}$

$\pi_V^H(X) = [\downarrow_H^G S^V, \downarrow_H^G X]^H$

$\underline{\pi}_V X =$



RO(G)-grading (Multiplicative Structure)

R ring in $Sp^G \rightsquigarrow \pi_V(R) \otimes \pi_W(R) \longrightarrow \pi_{V \oplus W}(R)$

R comm ring in $Sp^G \rightsquigarrow \pi_\star(R)$ graded-comm ⚠

Ex $G = C_2$ $\pi_{i+k\sigma}(R) \otimes \pi_{j+\ell\sigma}(R) \longrightarrow \pi_{i+j+(k+\ell)\sigma}(R)$

$$\boxed{\alpha \cdot \beta = (-1)^{ij} \varepsilon^{k\ell} \beta \cdot \alpha}$$

$\varepsilon = \tau w : S^\sigma \wedge S^\sigma \cong S^\sigma \wedge S^\sigma$

$S^0 \longrightarrow k\mathbb{R} \longrightarrow H_{C_2}\underline{\mathbb{Z}} \overset{\underline{\pi}_0}{\rightsquigarrow} \underline{A} \longrightarrow \underline{\mathbb{Z}} \cong \underline{\mathbb{Z}}$

$\varepsilon \longmapsto -1$

R ring in Sp^G

$\pi_V(R) \otimes \pi_W(R) \longrightarrow \pi_{V\oplus W}(R)$

$G = C_2$

$\alpha \in \pi_{i+k\sigma}(R)$

$\beta \in \pi_{j+l\sigma}(R)$

$\alpha \cdot \beta = (-1)^{ij} \varepsilon^{kl} \beta \cdot \alpha$

$S^0 \longrightarrow k\mathbb{R} \longrightarrow H_{C_2}\underline{\mathbb{Z}}$

$\varepsilon \longmapsto -1$

RO(G)-grading

Some elements

- $V \in Rep(G)$, $S^0 \xrightarrow{a_V} S^V$ $\quad a_V \in \pi_{-V}(S^0)$
- For $\sigma \in Rep(C_2)$ $\quad C_{2+} \to S^0 \xrightarrow{a} S^\sigma \Rightarrow$ $\boxed{\mathrm{im}(T_e^{C_2}) = \ker(a)}$

  $S^{-\sigma} \xrightarrow{\Sigma^{-\sigma} a} S^0 \to C_{2+} \Rightarrow$ $\boxed{\ker(R_e^{C_2}) = \mathrm{im}(a)}$
- $V \in Rep(G)$, $u_V \in \pi_{V-V} H_G\underline{\mathbb{Z}} \cong \tilde{H}(S^{V-H}; \underline{\mathbb{Z}})$,

  $V = \dim V$ $\quad$ if V orientable, $R_e^G u_V = \pm 1$.
- $G = C_2$ $\quad 2\sigma$ orientable $\Rightarrow \exists\ u_{2\sigma} \in \pi_{2-2\sigma} H_{C_2}\underline{\mathbb{Z}}$

$$S^0 \xrightarrow{a} S^\sigma$$

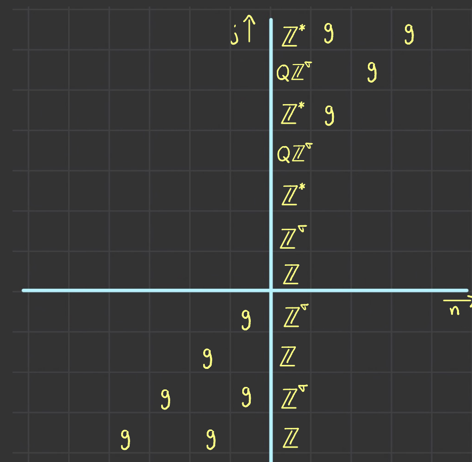
$$\boxed{\begin{aligned} &\mathrm{im}(T_e^{C_2}) = \ker(a) \\ &\ker(R_e^{C_2}) = \mathrm{im}(a) \end{aligned}}$$

$$u_{2\sigma} \in \pi_{2-2\sigma} H_{C_2}\underline{\mathbb{Z}}$$

$$R u_{2\sigma} = \pm 1 \in \pi_0 H\mathbb{Z}$$

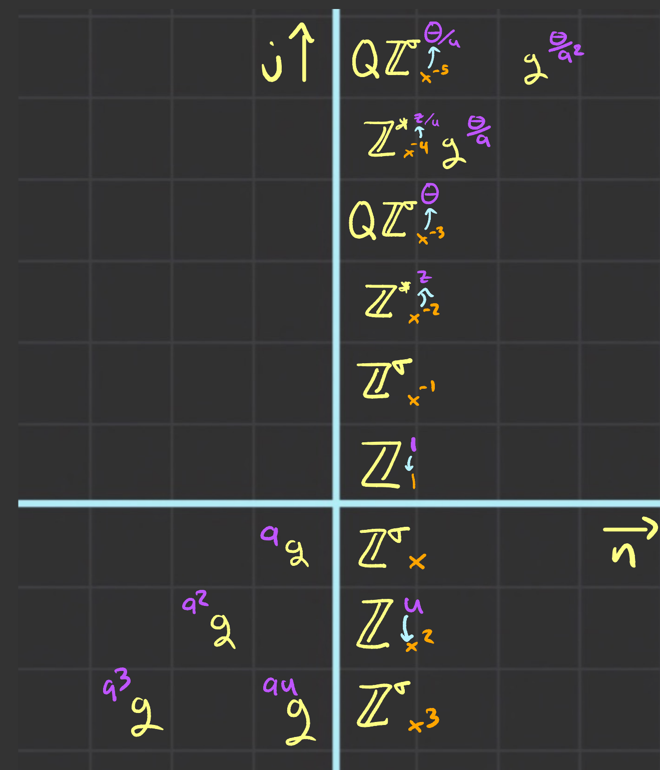
RO(G)-grading $\underline{\pi}_\star H_{C_2}\underline{\mathbb{Z}}$

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$



$g = \begin{smallmatrix}\mathbb{Z}/2 \\ 0\end{smallmatrix}$ $\quad \underline{\mathbb{Z}} = \begin{smallmatrix}\mathbb{Z} \\ 1 \downarrow \uparrow 2 \\ \mathbb{Z}\end{smallmatrix}$ $\quad \underline{\mathbb{Z}}^* = \begin{smallmatrix}\mathbb{Z} \\ 2 \downarrow \uparrow 1 \\ \mathbb{Z}\end{smallmatrix}$

$\underline{\mathbb{Z}}^\sigma = \begin{smallmatrix}0 \\ \\ \mathbb{Z}_-\end{smallmatrix}$ $\quad Q\underline{\mathbb{Z}} = \begin{smallmatrix}\mathbb{Z}/2 \\ \uparrow \\ \mathbb{Z}_-\end{smallmatrix}$



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RO(G)-grading: The slice spectral sequence

$$E_2^{s,t} = \underline{\pi}_{t-s} P_t^t X \Longrightarrow \underline{\pi}_{t-s} X.$$

$$d_r : \underline{\pi}_n P_t^t X \longrightarrow \underline{\pi}_{n-1} P_{t+r-1}^{t+r-1} X$$

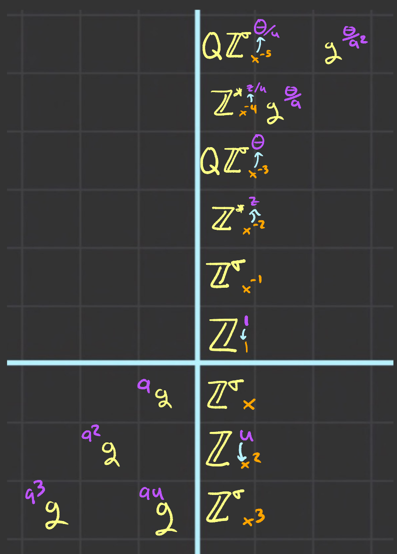
$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$

g: $\begin{matrix}\mathbb{Z}/2\\ 0\end{matrix}$ $\quad \underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}\\ 1\downarrow\uparrow 2\\ \mathbb{Z}\end{matrix}$ $\quad \underline{\mathbb{Z}}^* = \begin{matrix}\mathbb{Z}\\ 2\downarrow\uparrow 1\\ \mathbb{Z}\end{matrix}$

$\underline{\mathbb{Z}}^\sigma = \begin{matrix}0\\ \mathbb{Z}_-\end{matrix}$ $\quad Q\underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}/2\\ \uparrow\\ \mathbb{Z}_-\end{matrix}$

RO(G)-grading: The slice spectral sequence

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$



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$\underline{\mathbb{Z}}^\sigma = \begin{matrix}0\\ \mathbb{Z}_-\end{matrix}$ $\quad Q\underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}/2\\ \uparrow\\ \mathbb{Z}_-\end{matrix}$

$$E_2^{s,V} = \underline{\pi}_{V-s} P_V^V X \Longrightarrow \underline{\pi}_{V-s} X.$$

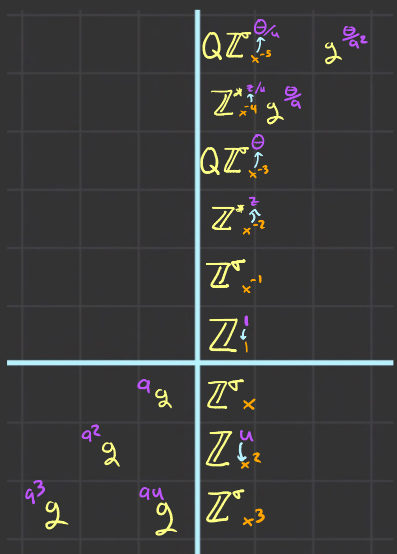
$$d_r: \underline{\pi}_{V-s} P_V^V X \longrightarrow \underline{\pi}_{V-s-1} P_{V+r-1}^{V+r-1} X$$

R comm ring in Sp^G

$$E_2^{s,V} \otimes E_2^{s',V'} \longrightarrow E_2^{s+s',V+V'}$$

$$\underline{\pi}_{V-s} P_V^V R \otimes \underline{\pi}_{V'-s'} P_{V'}^{V'} R \longrightarrow \underline{\pi}_{V+V'-s-s'} P_{V+V'}^{V+V'} R$$

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$



$g: \begin{matrix}\mathbb{Z}/2\\ 0\end{matrix}$ $\quad \underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}\\ {}^{1}\downarrow\uparrow^{2}\\ \mathbb{Z}\end{matrix}$ $\quad \underline{\mathbb{Z}}^{*} = \begin{matrix}\mathbb{Z}\\ {}^{2}\downarrow\uparrow^{1}\\ \mathbb{Z}\end{matrix}$

$\underline{\mathbb{Z}}^{\sigma} = \begin{matrix}0\\ \mathbb{Z}_{-}\end{matrix}$ $\quad Q\underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}/2\\ \uparrow\\ \mathbb{Z}_{-}\end{matrix}$

RO(G)-grading: The slice spectral sequence

$$E_2^{s,V} = \underline{\pi}_{V-s} P_V^V X \Longrightarrow \underline{\pi}_{V-s} X.$$

$$d_r : \underline{\pi}_{V-s} P_V^V X \longrightarrow \underline{\pi}_{V-s-1} P^{V+r-1}_{V+r-1} X$$

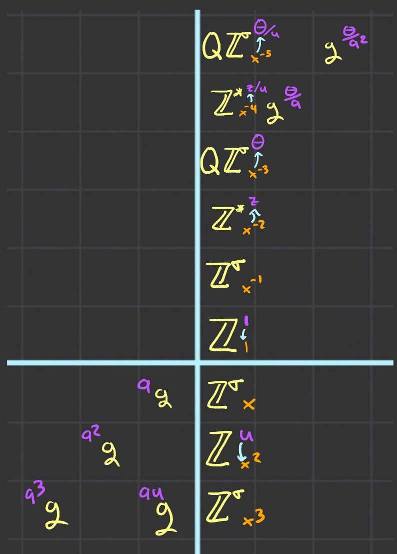
R comm ring in Sp^G

$$E_2^{s,V} \otimes E_2^{s',V'} \longrightarrow E_2^{s+s',V+V'}$$

$$\underline{\pi}_{V-s} P_V^V R \otimes \underline{\pi}_{V'-s'} P_{V'}^{V'} R \longrightarrow \underline{\pi}_{V+V'-s-s'} P^{V+V'}_{V+V'} R$$

$RO(C_2)$-graded SSS for $k\mathbb{R}$

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$



$g = \begin{smallmatrix}\mathbb{Z}/2\\ 0\end{smallmatrix}$ $\quad \underline{\mathbb{Z}} = \begin{smallmatrix}\mathbb{Z}\\ 1\,\downarrow\uparrow\,2\\ \mathbb{Z}\end{smallmatrix}$ $\quad \underline{\mathbb{Z}}^* = \begin{smallmatrix}\mathbb{Z}\\ 2\,\downarrow\uparrow\,1\\ \mathbb{Z}\end{smallmatrix}$

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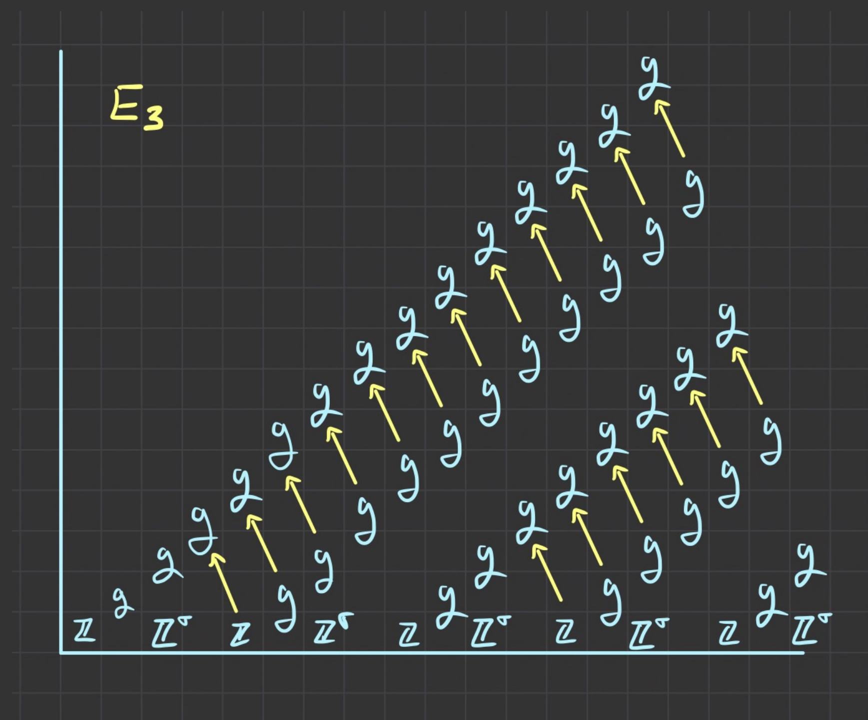


$RO(C_2)$-graded SSS for $k\mathbb{R}$

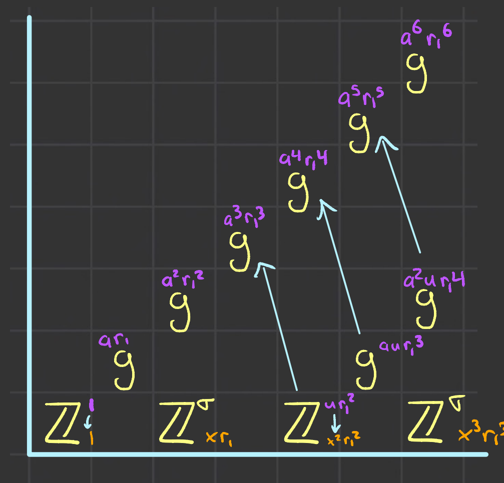
$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$

$g = \begin{smallmatrix}\mathbb{Z}/2\\ 0\end{smallmatrix}$ $\underline{\mathbb{Z}} = \begin{smallmatrix}\mathbb{Z}\\ 1\downarrow\uparrow 2\\ \mathbb{Z}\end{smallmatrix}$ $\underline{\mathbb{Z}}^* = \begin{smallmatrix}\mathbb{Z}\\ 2\downarrow\uparrow 1\\ \mathbb{Z}\end{smallmatrix}$

$\underline{\mathbb{Z}}^{\sigma} = \begin{smallmatrix}0\\ \mathbb{Z}_{-}\end{smallmatrix}$ $Q\underline{\mathbb{Z}} = \begin{smallmatrix}\mathbb{Z}/2\\ \uparrow\\ \mathbb{Z}_{-}\end{smallmatrix}$

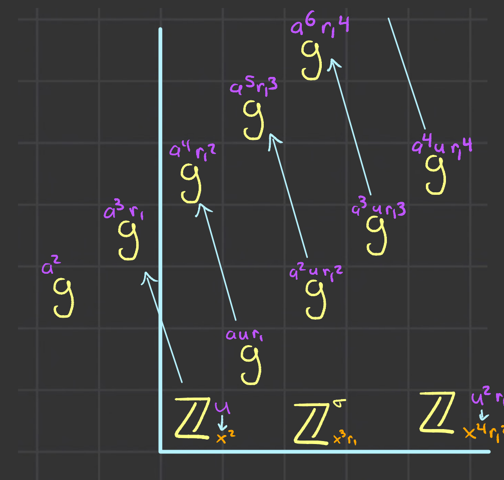
$$\underline{\pi}_\star \Sigma^{\rho} H_{C_2}\underline{\mathbb{Z}} \cong \underline{\pi}_\star H_{C_2}\underline{\mathbb{Z}}\{r_1\} \qquad r_1 \in \pi_\rho \Sigma^{\rho} H_{C_2}\underline{\mathbb{Z}}$$

$E_3^{*,*}$



$$d_3(u r_1^2) = a^3 r_1^3$$

$E_3^{*,*+2-2\sigma}$



$$d_3(u) = a^3 r_1$$

3

$RO(C_2)$-graded SSS for $k\mathbb{R}$

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$

$g = \begin{matrix}\mathbb{Z}/2\\ 0\end{matrix}$ $\quad \underline{\mathbb{Z}} = \begin{matrix}\mathbb{Z}\\ 1\downarrow\uparrow 2\\ \mathbb{Z}\end{matrix}$ $\quad \underline{\mathbb{Z}}^* = \begin{matrix}\mathbb{Z}\\ 2\downarrow\uparrow 1\\ \mathbb{Z}\end{matrix}$

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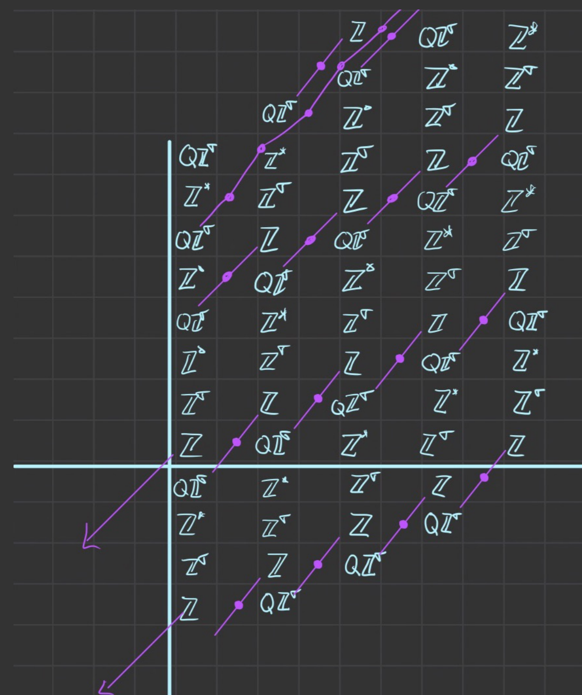
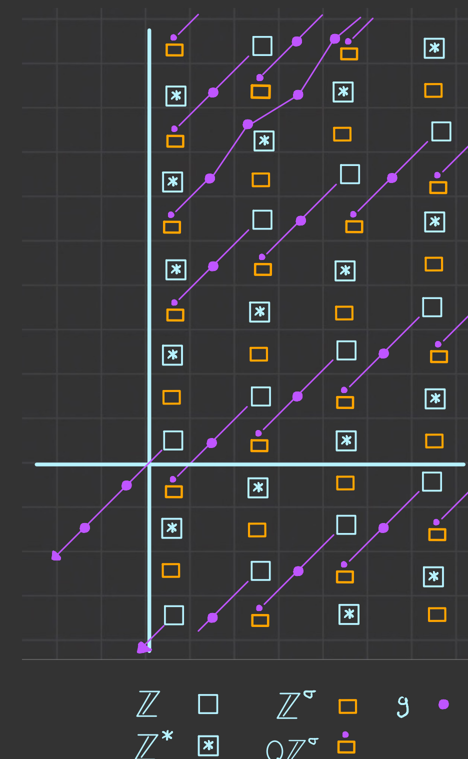


$RO(C_2)$-graded SSS for $k\mathbb{R}$

$\underline{\pi}_{n-j+j\sigma} H_{C_2}\underline{\mathbb{Z}}$

Geometric fixed points

$$\Phi^{C_2}(k\mathbb{R}) \simeq H\mathbb{F}_2[u^2]$$



Lecture 4

- Duality
- Norms
- MUIR, BPIR