1. Show that $\Sigma^n H C^2 g \simeq H C^2 g$, and use this to compute $\Phi^C H C^2 g$.

2. Show that $\Phi^C X \simeq \Phi^{C_4 / C_2} X^{C_2}$. (Hint: show that, if $p: C_4 \longrightarrow C_4 / C_2$ denotes the quotient map, then $p^* \widetilde{EP}_{C_2}$ is a model for $\widetilde{EP}_{C_4}$.)

3. For $G = C_2$, show that a map of $C_2$-spectra $f: X \longrightarrow Y$ is a $\pi_*$-isomorphism if and only if it yields an isomorphism of $RO(C_2)$-graded homotopy groups. (Hint: Use the cofber sequence $C_2^+ \rightarrow S^0 \rightarrow S^\varnothing$.)

4. $(C_4$-slice towers$)$

(a) Use Figure 3 of HHR’s article on the $C_4$-analogue of real $K$-theory to show that $\Sigma^n H C^4 Z$ is an $n$-slice for $0 \leq n \leq 4$.

(b) The $C_4$-Mackey functor $Z(2,1)$ is displayed to the right. Show that there is a short exact sequence of $C_4$-Mackey functors $Z(2,1) \hookrightarrow Z \longrightarrow g = C^*_4 Z / 2$.

(c) Show that $P_{8} = \Sigma^2 H C^4 g \longrightarrow \Sigma^5 H C^4 Z \longrightarrow \Sigma^{3+2\sigma} H C^4 Z = P_{5}^G$ is the slice tower for $\Sigma^5 H C^4 Z$. (Hint: Figure 6 of the $C_4$-article of HHR tells you that $\Sigma^{2-2\sigma} H C^4 Z \simeq H C^4 Z(2,1)$.)

5. We described the Mackey functors $\pi_0 K R$, for $n \geq 0$. This then yields the Mackey functors $\pi_n K R$, for $n \in \mathbb{Z}$, using the 8-fold periodicity of $K R$. Use the $\rho$-periodicity of $K R$ to write down the $RO(C_2)$-graded Mackey functors $\pi_*$ $K R$. Display these in a chart, and display the nontrivial $a$-multiplications.

6. Use the equivalences $HQ Z^\varnothing \simeq \Sigma^{3-3\sigma} H Z$ and $H Z^* \simeq \Sigma^{2-2\sigma} H Z$ to show that $I Z HQ Z^\varnothing \simeq \Sigma^{\varnothing-1} H Z$. 