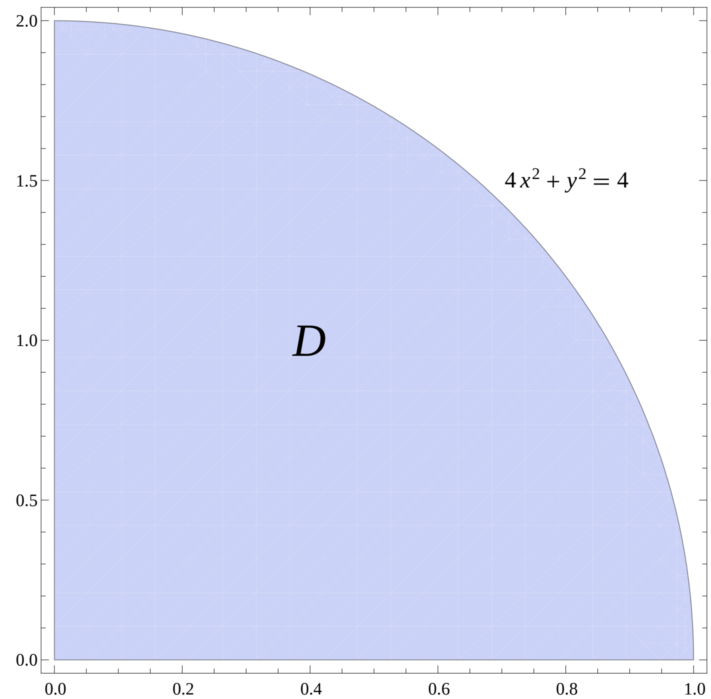
Math 241 - Quiz 4 Solutions - Tuesday, November 1

1. Let D be the region in the plane bounded by $x \geq 0$, $y \geq 0$, and the ellipse $4x^2 + y^2 = 4$ and let R be the solid lying above D and below the graph of the function $f(x, y) = x + 2y$.



(a) Set up, but do not evaluate, the two double integrals in rectangular coordinates that calculate the volume of R. (3 points)

SOLUTION:

$$\int_0^2 \int_0^{\sqrt{4-y^2}/2} x + 2y \, dx \, dy$$

$$\int_0^1 \int_0^{2\sqrt{1-x^2}} x + 2y \, dy \, dx$$

(b) Set up, but do not evaluate, an integral in polar coordinates that calculates the volume of R. (2 points)

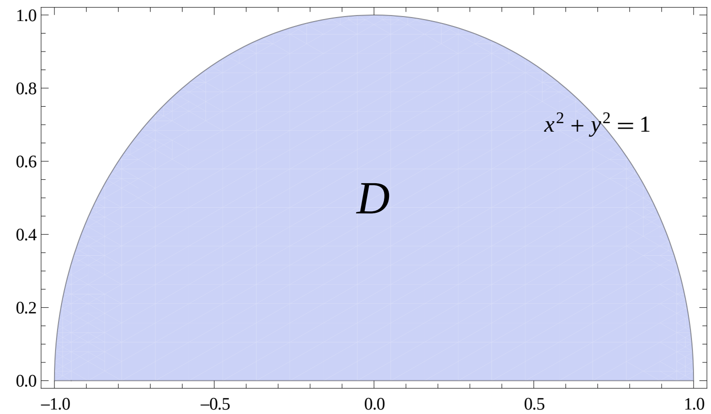
SOLUTION:

Converting the equation $4x^2 + y^2 = 4$ to polar we obtain $r^2(4\cos^2\theta + \sin^2\theta) = 4$ or $r^2(3\cos^2\theta + 1) = 4$. Solving for r we have $r = \frac{2}{\sqrt{1+3\cos^2\theta}}$. Thus remembering the factor of r that comes in when we convert to polar we have:

$$\int\int_D x + 2y \, dA = \int_0^{\pi/2} \int_0^{\frac{2}{\sqrt{1+3\cos^2\theta}}} (r\cos\theta + 2r\sin\theta) r \, dr \, d\theta$$

2. Let D be the region in the plane bounded below by the x-axis and above by the circle $x^2+y^2=1$. Convert the double integral $\iint_D x+2\;dA$ into polar coordinates and evaluate the double integral. (3 points)

SOLUTION:



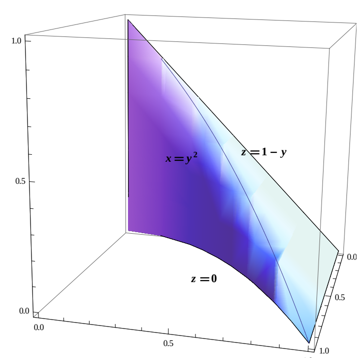
$$\int\int_D x+2\,dA = \int_0^{\pi}\int_0^1 (r\cos\theta+2)r\,dr\,d\theta = \pi$$

3. Rewrite the given triple integral in the two orders $dx\,dy\,dz$ and $dz\,dx\,dy$:

$$\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx.$$

(2 points)

SOLUTION:



$$\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx = \int_0^1\int_0^{1-z}\int_0^{y^2} f(x,y,z)\,dx\,dy\,dz$$

$$\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx = \int_0^1\int_0^{y^2}\int_0^{1-y} f(x,y,z)\,dx\,dy\,dz$$