SOLUTIONS TO WORKSHEET FOR THURSDAY, OCTOBER 20, 2011

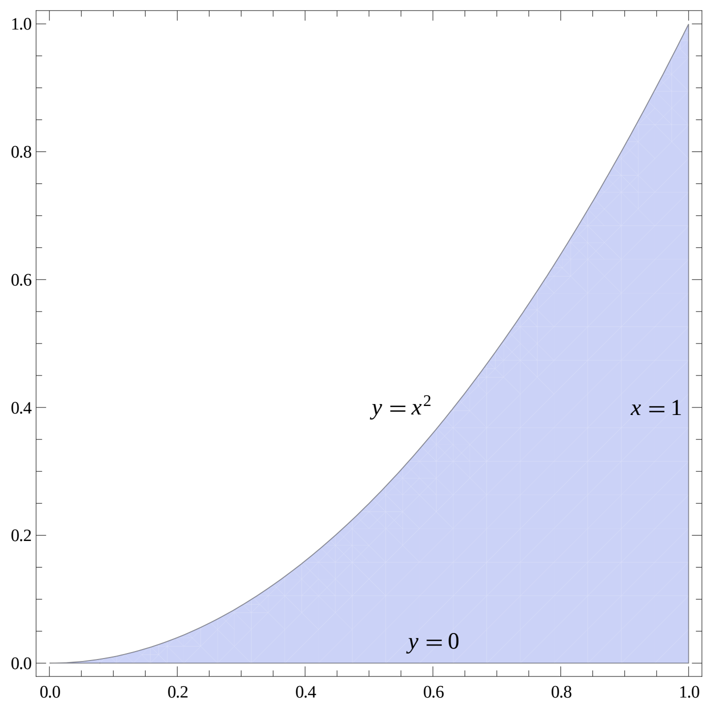
1. Evaluate the following integral by reversing the order of integration:

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy.$$

(Hint: When you change to $dx\, dy$, be sure to also change the bounds of integration.)

SOLUTION:

We are integrating over the region below:



Changing the order of integration we get

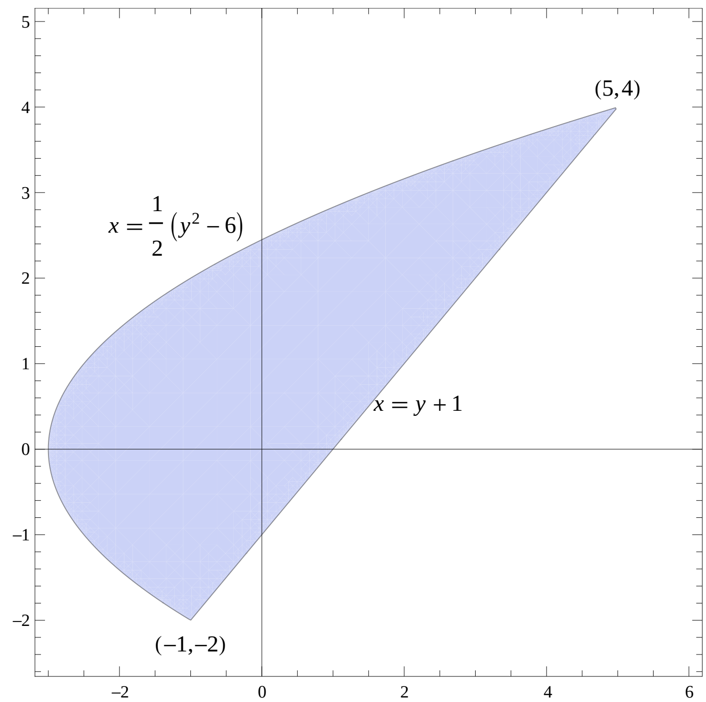
$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy = \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\, dy\, dx$$

$\int_0^1 \int_0^{x^2} \sqrt{x^3+1}\, dy\, dx = \int_0^1 x^2\sqrt{x^3+1}\, dx = 1/3[(x^3+1)^{3/2}]_0^1 = 1/3(2^{3/2}-1).$

2. Consider the region bounded by the curves determined by $-2x + y^2 = 6$ and $-x + y = -1$.

(a) Sketch the region R in the plane.

SOLUTION:



(b) Set up and evaluate an integral of the form $\iint_R dA$ that calculates the area of R.

SOLUTION:

$$\int_{-2}^{4} \int_{\frac{y^2-6}{2}}^{y+1} dx\, dy = \int_{-2}^{4} y + 1 - \frac{y^2-6}{2}\, dy = \left[-1/6y^3 + 1/2y^2 + 4y\right]_{-2}^{4} = 18$$

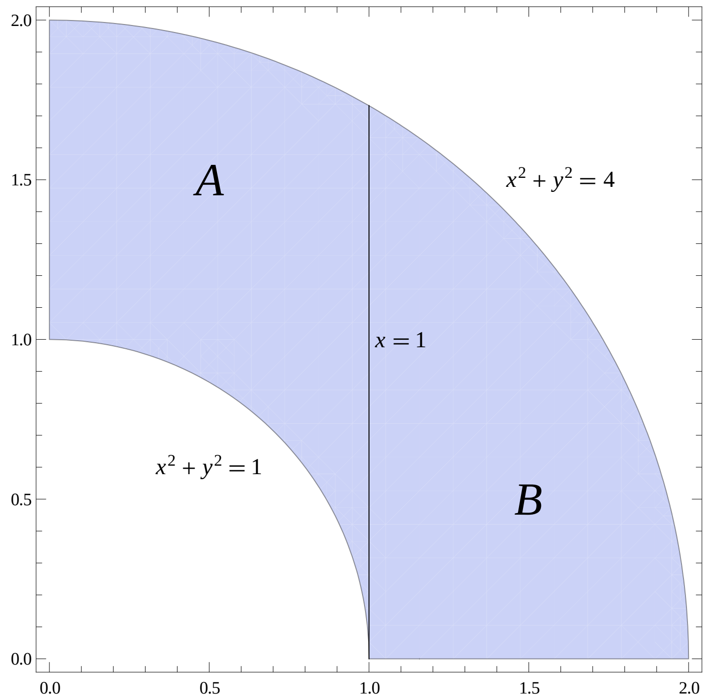
3. Consider the region R in the first quadrant which lies above the x-axis, to the right of the y-axis and between the circles of radius 1 and 2 centered at $(0,0)$. *Without using polar coordinates*, evaluate

$$\iint_R y\,dA.$$

Hint: You'll have to break R into several simple (Type I and II) regions.

SOLUTION:

Break up the region into two as shown below:



$$\iint_R y\,dA = \iint_A y\,dA + \iint_B y\,dA = \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} y\,dy\,dx + \int_1^2 \int_0^{\sqrt{4-x^2}} y\,dy\,dx$$

$$= \int_0^1 \left[y^2/2\right]_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} dx + \int_1^2 \left[y^2/2\right]_0^{\sqrt{4-x^2}} dx = \int_0^1 3/2\,dx + \int_1^2 1/2(4-x^2)\,dx$$

$$= 7/3$$

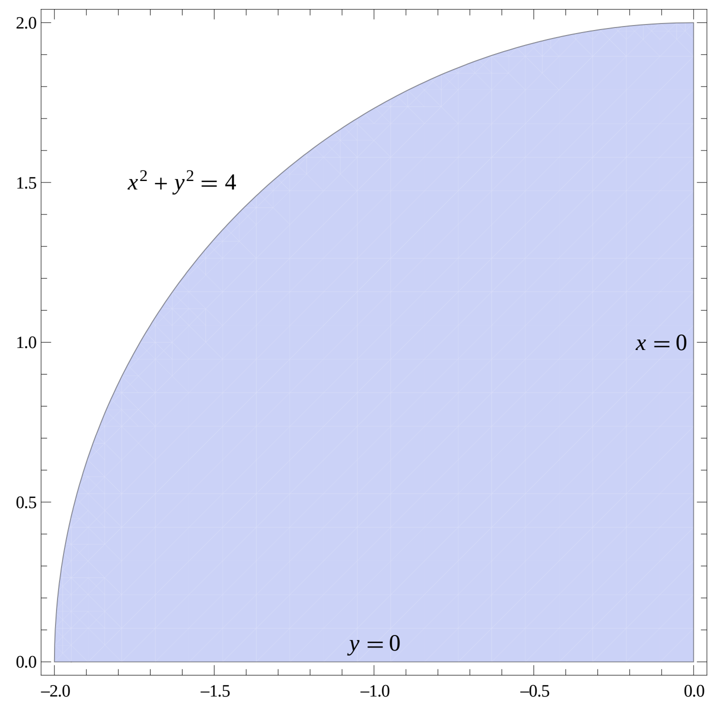
4. Evaluate

$$\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} (x^2+y^2)\,dy\,dx.$$

Hint: don't do it directly.

SOLUTION:

The region over which we are integrating is:



Converting to polar we get

$$\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} (x^2+y^2)\,dy\,dx = \int_{\pi/2}^{\pi}\int_{0}^{2}(r^2)r\,dr\,d\theta = 2\pi$$