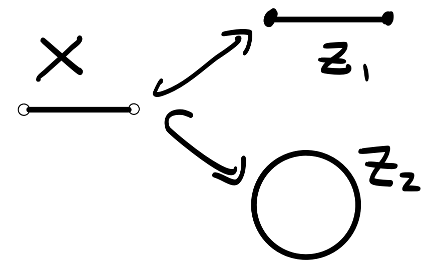
Group: ______________ ______________ ______________ Name: _______________

Math 351 - Elementary Topology
Wednesday, December 5 ∗∗ *One-point Compactification*

A **compactification** of a space X is a compact space Z together with an embedding $X \hookrightarrow Z$ such that X is dense in Z. Pictured to the right are two compactifications of an open interval.



Given any space X, define a new space $X_+ = X \cup \{\infty\}$ as follows:

- every open subset $U \subseteq X$ is considered open in X_+
- The neighborhoods V of the new point ∞ are the subsets such that $X \setminus V$ is *compact*.

It can be shown that if X is Hausdorff then this defines a topology on X_+, and it is clear that X is a subspace of X_+.

1. Assume the above defines a topology on X_+. Show that X_+ is compact.
2. What space is $\mathbb{R}_+$? What are $\mathbb{R}^2_+$ and $\mathbb{R}^n_+$?
3. What is $\Big[(0,1) \cup (2,3)\Big]_+$?
4. Assume X is **not** compact. Show that X is dense in X_+. What is another description of X_+ if X is already compact?

Write your answer(s) on the rest of this sheet (and back).
