Group: ______________ ______________ ______________ Name: ______________

Math 351 - Elementary Topology
Wednesday, October 3 $**$ *Continuous functions*

The problems below concern continuous functions. Recall that $f : X \longrightarrow Y$ is continuous if for every open set $U \subset Y$, the preimage $f^{-1}(U)$ is open in X. Make sure to justify all of your answers.

1. (2 **points**) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x & x \geq 0 \\ x - 1 & x < 0. \end{cases}$$

   Show that f is continuous when considered as a function

$$f : \mathbb{R}_{\ell\ell} \longrightarrow \mathbb{R}.$$

   ($\mathbb{R}_{\ell\ell}$ is $\mathbb{R}$ with the "lower-limit", or half-open, topology.) Feel free to use the result from class that f is continuous if and only if it is continuous at every $c \in \mathbb{R}_{\ell\ell}$.

2. (**2 points**) Let $f, g : X \longrightarrow Y$ be continuous, and suppose that Y is Hausdorff. Show that if $D \subset X$ is *dense* in X and $f(d) = g(d)$ for all $d \in D$, then necessarily $f(x) = g(x)$ for all $x \in X$. In other words, show that if f and g agree on a dense subset, then they agree everywhere.

Write your answer(s) on the rest of this sheet (and back).
