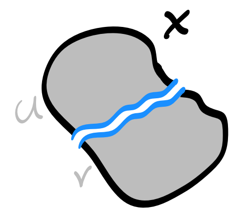
Group: ____________ ____________ ____________ Name: _____________

Math 351 - Elementary Topology
Wednesday, November 28 ** *Connectedness*

The following are equivalent ways of stating that a space X is **connected**



- The only nonempty closed and open subset $U \subseteq X$ is X itself.
- If $X = U \amalg V$ with U and V both open, then either U or V is empty.

Define an equivalence relation $\sim$ on X by $x \sim y$ if there exists a *connected* subset of X that contains both x and y.
The equivalence class $\overline{x}$ of $x \in X$ is called the "connected component" of x in X.

1. Show that the relation defined above is transitive.
2. Show that a "connected component" is in fact connected.
3. Show that $\mathbb{R}_{fc}$ has only one component (in other words, show it is a connected space).
4. Find an example of a space X and a connected component $C \subset X$ such that C is *not* open in X. (The components are always closed, however.)

Write your answer(s) on the rest of this sheet (and back).
