Math 351 - Elementary Topology
Friday, December 7 $**$ *Final Exam Review Problems*

1. Given a space X and points x and y in X, a **path** from x to y is a continuous map $\gamma : [0,1] \longrightarrow X$ such that $\gamma(0) = x$ and $\gamma(1) = y$. Show that if every pair of points in X can be connected by a path, then X is connected.

2. Define a subset $X \subseteq \mathbb{R}^2$ as

$$X = \left\{ (x, \sin(1/x) \mid x \in \left(0, \frac{2}{\pi}\right] \right\}$$

and let $Z = X \cup \Big(\{0\} \times [-1,1]\Big)$.

   (a) Show that Z is connected.

   (b) **Challenge problem:** Show that there is no path from $(0,0)$ to $\left(\frac{2}{\pi}, 1\right)$ in Z.

3. In $\mathbb{R}_{\ell\ell}$, show there is no (continuous) path from 0 to 1.

4. Show that the punctured plane $\mathbb{R}^2 \setminus \{(0,0)\}$ is connected. Use this to show that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

5. Prove that S^1 is not homeomorphic to $\mathbb{R}$.

6. Prove that neither S^1 nor $\mathbb{R}$ are homeomorphic to S^2.

7. Prove that neither S^1 nor $\mathbb{R}$ are homeomorphic to the torus T^2.

8. Show that $[0,1]$ is not compact in the lower limit topology.

9. Consider $A = \{1/n \mid n \in \mathbb{N}\}$. This is homeomorphic to the subset

$$B = \left\{ \left(\cos\left(\frac{\pi}{n}\right), \sin\left(\frac{\pi}{n}\right) \right) \mid n \in \mathbb{N} \right\} \subseteq S^1.$$

Let $C = \overline{B}$ be the closure of B in S^1. Describe the topology on C and find all connected components of C.