

MA671–001 Complex Analysis
Spring 2020
Problem Set 4
DUE: Friday, 6 March 2020

1. Stein and Shakarchi, page 64: Problem 1.
2. Harmonic functions and harmonic extensions.
 - (a) Stein and Shakarchi, page 66: Problem 11.
 - (b) Stein and Shakarchi, page 67: Problem 12 b (we already did part a).
 - (c) A comment: Suppose $f(\varphi)$ is a continuous (therefore periodic) function on the unit circle. Then $u_f(z)$, with $z = re^{-i\theta}$, $0 \leq r < 1$, defined on the unit disc by

$$u_f(z) = \frac{1}{2\pi} \int_0^{2\pi} P_r(\theta - \varphi) f(\varphi) d\varphi$$

is harmonic and satisfies the boundary condition $u_f|_{\mathcal{C}} = f$. This function u_f is called the harmonic extension of f to the disc (nothing for you to compute here).

3. Stein and Shakarchi, page 103: Problem 2.
4. Stein and Shakarchi, page 103: Problem 3.