**MA671–001 Complex Analysis**
**Spring 2020**
**Problem Set 5**
**DUE: 1 April 2020**

1. Find the maximum of the modulus of $f(z) = e^{z^2}$ in the unit disk.

2. Let $f : \mathcal{A} \to C$ be analytic and suppose $f'(z) \neq 0$ on $\mathcal{A}$. Suppose $z_0 \in \mathcal{A}$ and $f(z_0) \neq 0$. Then, given $\epsilon > 0$ small, there exist two points $z_1, z_2 \in B_\epsilon(z_0)$ so that $|f(z_2)| > |f(z_0)|$ and $|f(z_1)| < |f(z_0)|$.

3. Evaluate the following contour integral:

$$\int_{|z|=1} \frac{\sin(e^{z^2})}{z^2}\, dz$$

4. Let $f : \mathcal{A} \to C$ be analytic and $\mathcal{A} \subset \mathcal{C}$ is open, connected, and bounded. Suppose there is a point $z_0 \in \mathcal{A}$ so that $|f(z)| \leq |f(z_0)|$ for all $z \in \mathcal{A}$. Then $f$ is a constant on $\mathcal{A}$.

5. Find the residue of the function at the point indicated: $f(z) = \frac{e^{z^2}}{(z-1)^2}$ at $z_0 = 1$.

6. Evaluate the following itegral:

$$\int_{\theta=0}^{2\pi} \frac{d\theta}{1 + a^2 - 2a\cos\theta},$$

for real $a > 0$ and $a \neq 1$. HINT: map from the interval $[0, 2\pi]$ to $z$ by setting $z = e^{i\theta}$.

7. Let $f : \mathcal{A} \to C$ be analytic and non-zero in a region $\mathcal{A}$. Then $f$ has no strict local minimum in $\mathcal{A}$.