MA162: Finite mathematics

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January 19, 2010

SCHEDULE:

- HW A0 is due Wednesday, Jan 20th, 2010 (tomorrow).
- HW A1 is due Monday, Jan 25th, 2010.
- HW A2 is due Monday, Feb 1st, 2010.
- HW A3 is due Sunday, Feb 7th, 2010.
- Exam 1 is Monday, Feb 8th, 5:00pm-7:00pm.

Today we will cover linear functions and mathematical models.

Mathclass.org

- Homework is turned in on the **Mathclass** website
- Time is nearly up for the first assignment
- Most people have completed it; don't fall behind, especially not this early
- A brief (anonymous) **survey** might help the class run more smoothly.
- There will be another survey at mid-term to let the instructor know how things are going

Linear functions

- **Functions** give a unique y value for every x
- Sometimes y is called the output, and x the input
- $y = f(x)$ is read "y is f of x" and means that f gives y when it takes x
- For instance the function that doubles its input is $f(x) = 2x$
- When this function is given 3, it doubles it, and gives back 6, "$f(3) = 6$"
- When this function is given 14, it doubles it, and gives back 28, "$f(14) = 28$"

Variables versus unknowns

- The English words "wind" and "wind" look very similar, but mean different things ("moving air" ; "to twist rope")
- "x" sometimes means a general, unspecified number, **"variable"**
- "x" sometimes means a specific, but **unknown** number
- For instance "$f(x) = x^2$" usually means $f(1) = 1^2$, $f(2) = 2^2$, $f(3) = 3^2$, $f(-17) = (-17)^2$, $f(\xi) = \xi^2$, etc.
- But $x^2 = 10^2 + 11^2$ suggests x is an unknown, and we should solve for it, $x \approx 14.866$

Tables of values

- When we say $y = f(x)$ we mean that x and y are related
- x is allowed to be anything (any number), and for each x we get a y, $y = f(x)$
- For instance when $f(x) = 2x$ is the doubling function,

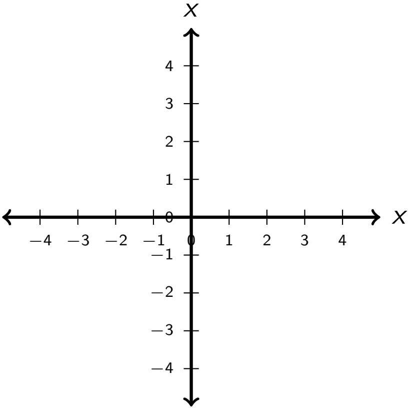
$$\begin{array}{ll} \text{for } x = 7, & \text{we get } y = 14, \\ \text{for } x = -5, & \text{we get } y = -10 \\ \text{for } x = \xi, & \text{we get } y = 2\xi \end{array}$$

x	y
7	14
-5	-10
ξ	2ξ

- We can make a **table of values** like above, or even better, we can draw a **graph**

Graphs of functions

- We can **draw** the function, that is, we draw its **graph**
- For each x, we find $y = f(x)$, then draw the point (x, y)
- For $f(x) = 2x$ we get the following:



x	**y**

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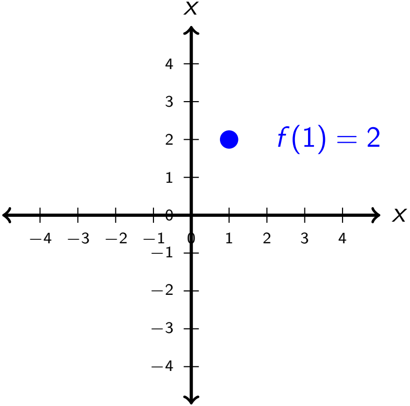

x	**y**
1	**2**

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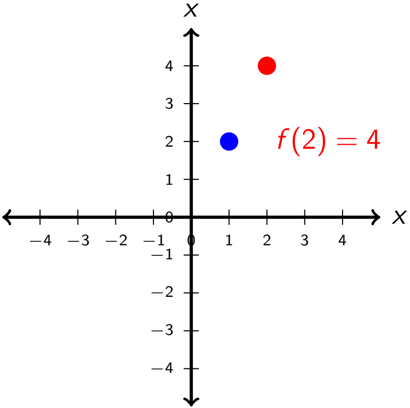

x	**y**
1	**2**
2	**4**

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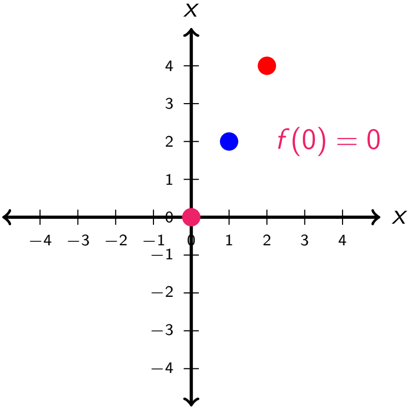

x	**y**
1	**2**
2	**4**
0	**0**

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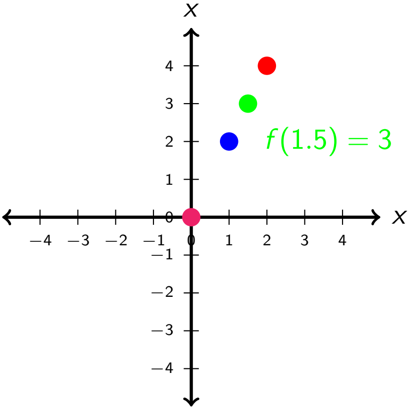

x	**y**
1	2
2	4
0	0
1.5	3

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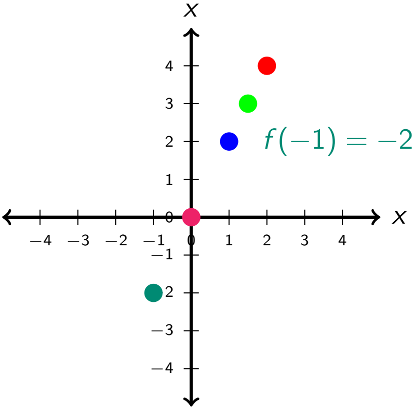

x	y
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2	4
0	0
1.5	3
-1	-2

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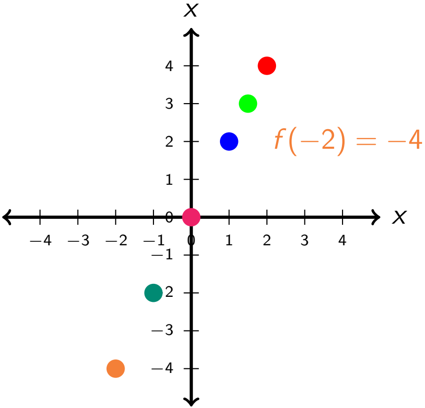

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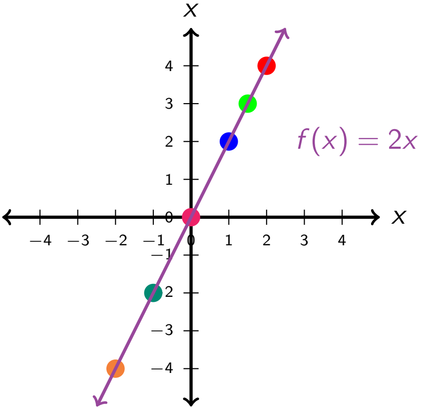

x	y
1	2
2	4
0	0
1.5	3
-1	-2
-2	-4
x	2x

Mathematical models

- Some aspects of life behave in a predictable manner
- We use numbers to measure these aspects, and equations relate the numbers to each other
- The phone company charges me in a predictable manner
- Every month they charge me \$50 plus five cents per text message
- If x is the number of text messages, then the bill (in dollars) is

$$f(x) = 50 + 0.05x$$

Production cost

- If I want to sell beaded bracelets at the county fair, then I need a few tools and the raw materials
- I can buy the tools for \$5, and for \$0.60 I can get enough beads and string for one bracelet
- The cost for me to make just one bracelet is \$5.60, but I could have made two for only \$6.20 (an extra \$0.60 for materials)
- If x is the number of bracelets I make, then the **cost** (in dollars) to make x bracelets is
$$C(x) = 5 + 0.6x$$

Revenue function

- If I decide to sell the bracelets for \$1 each,
  then if I sell 3 bracelets, I get \$3.
- If I sell x bracelets, then the **revenue** (in dollars) is:

$$R(x) = x$$

- What is interesting to me is the **profit**,

$$P(x) = R(x) - C(x) = x - (5 + 0.6x) = 0.4x - 5$$

x	C	R	P
1	5.60	1.00	-4.60
2	6.20	2.00	-4.20
3	6.80	3.00	-3.80
12	12.20	12.00	-0.20
13	12.80	13.00	0.20
20	17.00	20.00	3.00

Section 1.3

- The homework and exams will use words like: **cost function, revenue function, profit function, fixed costs, variable costs, supply equation, demand equation**
- The textbook, section 1.3, defines these words, and works example problems with them
- We will go over supply, demand, and market equilibrium on Thursday (section 1.4)
- You'll be asked to solve non-trivial problems that have you determine supply and demand equations from values given
- You'll be asked to convert temperature, but will be given the functions

Lab time: Activity for 1.3a

- Form groups of 1-4 people and begin working on activity 1.3a
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

More about cost functions

- If it costs \$10 to make fifty widgets and \$15 to make one hundred widgets, then the cost function is

$$C(x) = 5 + 0.1x$$

- $C(0) = 5$, even if you don't make any widgets, the **fixed cost** is \$5.
- If you have already paid for x, and want to make one more, then the **marginal cost** is

$$C(x+1) - C(x) = (5 + 0.1(x+1)) - (5 + 0.1x) = 0.1$$

- The marginal cost is another word for **slope**.
  The words **variable cost** are also used.

Linear functions

- If the marginal cost does not depend on how many you've already made, then the function is called **linear**
- A linear function looks like $f(x) = m \cdot x + b$
- A function like $g(x) = x^2$ is not linear:
  - $g(2) = 4$, $g(1) = 1$, so the marginal cost at $x = 1$ is $g(2) - g(1) = 4 - 1 = 3$
  - $g(11) = 121$, $g(10) = 100$, so the marginal cost at $x = 10$ is $g(11) - g(10) = 121 - 100 = 21$
- The majority of this course will focus on linear functions

Outsourcing the beaded bracelet selling

- You've decided to go big time and have other people sell your bracelets
- You've just watched the classic film *Newsies* and realize the best way to handle business is to have your "beadies" (sales people) pay you a low price for the bracelets, and then sell the bracelets at the county fair for a higher price
- Unfortunately, the market has changed since the 1920s and your beadies think they can tell you the price
- How many bracelets would you want to make if they are willing to pay you \$0.75 each?

The analysis of the beadie bargain

- Your cost function has not changed, $C(x) = 5 + 0.6x$
- Your revenue function has decreased to, $R(x) = 0.75x$
- Your profit function is therefore,

$$P(x) = R(x) - C(x) = 0.15x - 5$$

- You are pretty sure you want to make \$40 profit
- So you need to solve:

$$40 = P(x) = 0.15x - 5$$

This just means $45 = 0.15x$, so $x = 300$. That's a lot of bracelets.

Beadie little somethings

- All is well the first week, but your bracelets aren't selling as fast as the beadies would like
- They're now only willing to pay \$0.70 each
- Your profit function is now five cents less:

$$P(x) = 0.10x - 5$$

- You still want to make \$40 profit, so you solve

$$40 = P(x) = 0.10x - 5$$

This is just $45 = 0.10x$, so $x = 450$. That is a LOT of bracelets. Maybe it is time to change businesses.

Supply functions

- We noticed that the price you can sell your goods for has an effect on how many you are willing to supply.
- For an individual seller, the analysis is very similar to what we just did: as the price lowers, eventually the seller just quits.
- On the other hand, as the price increases, more people will decide to get into business.
- For the entire market, we get a nicer summary: the number of suppliers of a good increases with the price.
- We get a (roughly) linear **supply function**, $S(x) = m \cdot x + b$

Demand function

- From the consumer's point of view, the higher the price, the fewer they will want to buy.
- For the entire market, for a small enough price range, the **demand function** is linear
$$D(x) = m \cdot x + b$$
- For instance, if 20 people are willing to buy your beaded bracelets at \$1.00, but only 10 at \$2.00, then you can predict 15 will buy your beaded bracelets at \$1.50.
- If x is the price in dollars, then the number of customers is about:
$$D(x) = 30 - 10x$$
- Thursday we will discuss how supply and demand are related.

Pizza crust production: the flour function

- You notice that it takes about $3/8$ cups of flour to make a 6" pizza crust, $2/3$ cups of flour to make a 9" pizza crust, and $1\frac{1}{2}$ cups of flour to make a 12" pizza crust.
- Is the amount of flour required linear in the pizza size?
- Can you make a guess at the flour function, F(x)?

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- I've noticed that pizza prices tend to behave like $C(x) = m \cdot x^{3/2}$ somewhere in between the linear cost functions $C(x) = m \cdot x^1 + b$ and the flour function $F(x) = m \cdot x^2$

Quiz time:

- Please answer on the provided quiz form:
- Fill in a table of values and graph the function $f(x) = 2x + 1$
- Give the linear cost function if it costs \$10 to make ten widgets, and \$20 to make a fifty widgets.