MA162: Finite mathematics

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SCHEDULE:

- HW A1 is due Monday, Jan 25th, 2010.
- HW A2 is due Monday, Feb 1st, 2010.
- HW A3 is due Sunday, Feb 7th, 2010.
- Exam 1 is Monday, Feb 8th, 5:00pm-7:00pm.

Today we will cover intersections of lines.

Market equilibrium (an 18th century perspective)

- The higher the price, the less consumers want to buy.
- The higher the price, the more producers want to sell.
- If the price is too low, supplies run out,
  Black market forms (Nintendo Wii 2006-2007), price raises
- If the price is too high, surplus on shelves,
  GIANT LIQUIDATION SALES
- In a healthy economy, the price is at *equilibrium*:
- Low enough that consumers demand all that is produced
  High enough that producers supply all that is demanded

Market equilibrium with linear supply and demand

- The **supply function** $S(x)$ describes how many units producers supply at the price x
- The **demand function** $D(x)$ describes how many units consumers demand at the price x
- In a small enough range (and always in MA162), the supply and demand functions are **linear**
- Many problems about supply and demand can be solved as problems about **lines**

Market research as seen on the exams

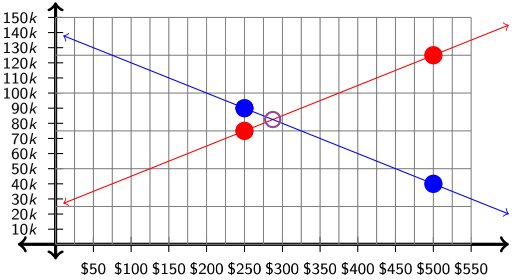
- We need to do market research in order to determine the supply and demand functions
- In this class, that means a word problem will tell us "At \$250 per Wii, 90000 units per day were demanded, and 75000 units per day were supplied. At \$500 per Wii, 40000 units per day were demanded, and 125000 units were supplied."
- In other words we have the following **table of values**:

x	S(x)	D(x)
250	75	90
500	125	40

where x is the price per Wii, $S(x)$ is how many thousands of Wii were supplied, and $D(x)$ is how many thousands of Wii were demanded

Graphing it

It can be very useful to graph the supply and demand functions. Just knowing two points is enough to graph the functions by hand.



x	S(x)	D(x)
250	75	90
500	125	40

The word problem

- "At \$250 per Wii, 90000 units per day were demanded, and 75000 units per day were supplied. At \$500 per Wii, 40000 units per day were demanded, and 125000 units were supplied. **Assuming linear supply and demand, at what price does the market reach equilibrium? How many Wii are demanded and supplied at this price?**"

- We begin by making a table of values and labelling our variables:

x	S(x)	D(x)
250	75	90
500	125	40

  where x is the price per Wii, $S(x)$ is how many thousands of Wii were supplied, and $D(x)$ is how many thousands of Wii were demanded

Solving for the equation of a line

- We need to find the supply and demand functions. We can do that by solving for the equation of line given two points on the line.
- **slope** is the change in y divided by the change in x

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

- The slope of a line **does not change** (marginal cost), so

$$\frac{y - y_2}{x - x_2} = m \text{ too}$$

- This gives **point-slope form**:

$$y - y_2 = m \cdot (x - x_2)$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$S(x) - S(250) = \frac{S(500) - S(250)}{500 - 250} \cdot (x - 250)$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$S(x) - S(250) = \frac{S(500) - S(250)}{500 - 250} \cdot (x - 250)$$

$$S(x) - 75 = \frac{125 - 75}{500 - 250} \cdot (x - 250)$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$S(x) - S(250) = \frac{S(500) - S(250)}{500 - 250} \cdot (x - 250)$$

$$\begin{aligned} S(x) - 75 &= \frac{125 - 75}{500 - 250} \cdot (x - 250) \\ &= \frac{50}{250}(x - 250) \end{aligned}$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$S(x) - S(250) = \frac{S(500) - S(250)}{500 - 250} \cdot (x - 250)$$

$$\begin{aligned} S(x) - 75 &= \frac{125 - 75}{500 - 250} \cdot (x - 250) \\ &= \frac{50}{250}(x - 250) = \frac{x}{5} - 50 \end{aligned}$$

Solving for the supply function

Now we solve for the supply equation, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$S(x) - S(250) = \frac{S(500) - S(250)}{500 - 250} \cdot (x - 250)$$

$$S(x) - 75 = \frac{125 - 75}{500 - 250} \cdot (x - 250)$$

$$= \frac{50}{250}(x - 250) = \frac{x}{5} - 50$$

$$S(x) = \frac{x}{5} + 25$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$D(x) - D(250) = \frac{D(500) - D(250)}{500 - 250} \cdot (x - 250)$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$D(x) - D(250) = \frac{D(500) - D(250)}{500 - 250} \cdot (x - 250)$$

$$D(x) - 90 = \frac{40 - 90}{500 - 250} \cdot (x - 250)$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$D(x) - D(250) = \frac{D(500) - D(250)}{500 - 250} \cdot (x - 250)$$

$$\begin{aligned} D(x) - 90 &= \frac{40 - 90}{500 - 250} \cdot (x - 250) \\ &= \frac{-50}{250} \cdot (x - 250) \end{aligned}$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$D(x) - D(250) = \frac{D(500) - D(250)}{500 - 250} \cdot (x - 250)$$

$$\begin{aligned} D(x) - 90 &= \frac{40 - 90}{500 - 250} \cdot (x - 250) \\ &= \frac{-50}{250} \cdot (x - 250) = \frac{-x}{5} + 50 \end{aligned}$$

Solving for the demand equation

Now we solve for the demand function, using the point-slope form:

x	S(x)	D(x)
250	75	90
500	125	40

$$y - y_2 = m \cdot (x - x_2)$$

$$D(x) - D(250) = \frac{D(500) - D(250)}{500 - 250} \cdot (x - 250)$$

$$\begin{aligned} D(x) - 90 &= \frac{40 - 90}{500 - 250} \cdot (x - 250) \\ &= \frac{-50}{250} \cdot (x - 250) = \frac{-x}{5} + 50 \end{aligned}$$

$$D(x) = \frac{-x}{5} + 140$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$S(x) = D(x)$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$S(x) = D(x)$$
$$\frac{1}{5}x + 25 = 140 - \frac{1}{5}x$$

Solving for the equilibrium price

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$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$S(x) = D(x)$$
$$\frac{1}{5}x + 25 = 140 - \frac{1}{5}x$$
$$\frac{1}{5}x + \frac{1}{5}x = 140 - 25$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$\begin{aligned} S(x) &= D(x) \\ \frac{1}{5}x + 25 &= 140 - \frac{1}{5}x \\ \frac{1}{5}x + \frac{1}{5}x &= 140 - 25 \\ \frac{2}{5}x &= 115 \end{aligned}$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$\begin{aligned} S(x) &= D(x) \\ \frac{1}{5}x + 25 &= 140 - \frac{1}{5}x \\ \frac{1}{5}x + \frac{1}{5}x &= 140 - 25 \\ \frac{2}{5}x &= 115 \\ x &= \frac{5}{2} \cdot 115 \end{aligned}$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$\begin{aligned} S(x) &= D(x) \\ \frac{1}{5}x + 25 &= 140 - \frac{1}{5}x \\ \frac{1}{5}x + \frac{1}{5}x &= 140 - 25 \\ \frac{2}{5}x &= 115 \\ x &= \frac{5}{2} \cdot 115 = 287.50 \end{aligned}$$

Solving for the equilibrium price

Now we have the supply and demand functions:

$$S(x) = \frac{1}{5}x + 25 \quad D(x) = 140 - \frac{1}{5}x$$

At market equilibrium, the number of Wii supplied is equal to the number of Wii demanded:

$$\begin{aligned} S(x) &= D(x) \\ \frac{1}{5}x + 25 &= 140 - \frac{1}{5}x \\ \frac{1}{5}x + \frac{1}{5}x &= 140 - 25 \\ \frac{2}{5}x &= 115 \\ x &= \frac{5}{2} \cdot 115 = 287.50 \end{aligned}$$

So the equilibrium price is \$287.50.

Solving for the equilibrium supply and demand

The equilibrium price is \$287.50, and the supply and demand functions are:

$$S(x) = \frac{1}{5}x + 25 \qquad D(x) = 140 - \frac{1}{5}x$$

Solving for the equilibrium supply and demand

The equilibrium price is \$287.50, and the supply and demand functions are:

$$S(x) = \frac{1}{5}x + 25 \qquad D(x) = 140 - \frac{1}{5}x$$

So the supply function there is

$$S(\$287.50) = 287.5/5 + 25 = 82.5$$

That is, 82,500 Wiis are supplied.

Solving for the equilibrium supply and demand

The equilibrium price is \$287.50, and the supply and demand functions are:

$$S(x) = \frac{1}{5}x + 25 \qquad D(x) = 140 - \frac{1}{5}x$$

So the supply function there is

$$S(\$287.50) = 287.5/5 + 25 = 82.5$$

That is, 82,500 Wiis are supplied.
The demand function there is

$$D(\$287.50) = 140 - 287.50/5 = 82.5$$

That is 82,500 Wiis are demanded.

Solving for the equilibrium supply and demand

The equilibrium price is \$287.50, and the supply and demand functions are:

$$S(x) = \frac{1}{5}x + 25 \qquad D(x) = 140 - \frac{1}{5}x$$

So the supply function there is

$$S(\$287.50) = 287.5/5 + 25 = 82.5$$

That is, 82,500 Wiis are supplied.
The demand function there is

$$D(\$287.50) = 140 - 287.50/5 = 82.5$$

That is 82,500 Wiis are demanded.

All is balanced.

Lab time: Activity 1.4a

- Form groups of 1-4 people and begin working on activity 1.4a
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

Break even analysis

- A basic business decision is to decide when a product is profitable.
- We have the **production cost function**, $C(x) = M \cdot x + F$ where M is the marginal cost of producing one *more* unit, and F is the fixed cost
- We have the **revenue function**, $R(x) = p \cdot x$ where p is the price at which we can sell each unit
- The difference is the **profit function**, $P(x) = R(x) - C(x)$
- To **break even** is to have $P(x) = 0$, That is, to have $R(x) = C(x)$
- If x is the break-even amount, then we must (produce and) sell at least x to break even

Break even analysis example

- If your fixed cost is \$5 and your marginal cost is \$0.60, then your cost function is $C(x) = 5 + 0.6x$
- If your marginal revenue is \$0.75, then your revenue function is $R(x) = 0.75x$
- The profit function is $P(x) = 0.75x - (5 + 0.6x) = 0.15x - 5$
- If we solve $0 = P(x) = 0.15x - 5$, then we get $x = 5/0.15 = 33.\overline{3} \approx 34$
- To break even you need to sell at least 34 beaded bracelets
- If the revenue drops to \$0.70 per bracelet,
  then your profit function drops to $P(x) = 0.1x - 5$,
  and you need to sell at least 50 beaded bracelets to break even

Cross-over point

- Beaded Bracelet King (or B.B. King as you are known in the trade) is investigating an alternative product
- The fixed cost is \$20 as it requires some much fancier tools, like needle-nose pliers and a vise
- The marginal cost is \$1.20 as it requires silver (plated) wire instead of string, and some fancier crystal beads
- However, according to your analysts on EBay, you can now expect a revenue of \$3 per bracelet
- Which is more profitable, your original design selling at \$1 each, or your new design?

Cross-over point analysis

- Your new product looks like:
  $C_n(x) = 20 + 1.2x$, $R_n(x) = 3x$, $P_n(x) = 1.8x - 20$
- Your old product looks like:
  $C_o(x) = 5 + 0.6x$, $R_o(x) = x$, $P_o(x) = 0.4x - 5$
- Make a table of values of the profit functions:

x	new	old
0	-20.00	-5.00
1	-18.20	-4.60
5	-11.00	-3.00
10	-2.00	-1.00
15	7.00	1.00

- By the time you are making any money at all,
  the new one is much better,
  but if they don't sell, you are at much higher risk

Where is the cross-over exactly?

- To find the exact value of x where the profit lines cross, we just solve:

$$\begin{aligned} P_n(x) &= P_o(x) \\ 1.8x - 20 &= 0.4x - 5 \\ 1.4x &= 15 \\ x &= 15/1.4 \approx 10.7 \end{aligned}$$

- So if you make 10 or fewer, you earn more profit with the old
  Well, you lose less money at least

- If you make 11 or more, you earn more profit with the new design

Lab time: Activity 1.4b

- Form groups of 1-4 people and begin working on activity 1.4b
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

Quiz: 1.4

- Fill in the table of values and graph $f(x) = 2x - 1$ and $g(x) = 5 - 2x$. Where do the graphs intersect?

- If a producer will supply 280 bushels of corn at \$2 per bushel and 400 bushels of corn at \$5 per bushel, then what is that producer's supply function?