MA162: Finite mathematics

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February 2, 2010

Schedule:

- HW A3 is due **Sunday**, Feb 7th, 2010.
  Common version solutions
- Exam 1 is Monday, Feb 8th, 5:00pm-7:00pm.
  Practice exam available
- HW B1 is due Monday, Feb 22nd, 2010.

Today we will cover 2.3 and pages 7-8 of the appendix: degeneracy and RREF

Appendix: Very efficiently solving systems

- We managed to solve some fairly big systems last time using our **new** algebra skills.
- Mostly it was repetitive, routine, soothing.
- But near the end we stopped the number pushing and revived the variables
- The quiz was easier, we just read the answers
- Today we learn to finish the easy way

Appendix: Cleaning above as well as below

- A matrix is in **REF** if no column (left of the bar) has two pivots
- This means that below and to the left of each pivot are zeros
- A matrix is in **RREF** if
  - it is in REF,
  - there are only zeros above pivots, and
  - pivots are equal to 1

Appendix: How to clean

- If a matrix is in REF, then a **possible target** is a non-zero number above a pivot
- We choose the right-most column with a possible target, and then choose the bottom-most possible target in that column
- The row operation is the same as before:

$$R_{target} - \frac{target}{active} \cdot R_{active}$$

- If a pivot is not equal to one, then we can divide the whole row by the pivot

Appendix: Example

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

RREF

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Lab time: Activity 2.3a

- Form groups of 1-4 people and finish working on activity 2.3a
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

2.3: What if things go wrong?

- Is this matrix in REF? RREF?

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- What could we do to fix it?
  - Row 2 can only make row 1 worse and vice versa!
  - Row 3 cannot do anything at all!
- Let's write it out in variables, and see what is going on:

$$x + y = 1 \qquad z = 1 \qquad 0 = 0$$

- Well that is not too bad? $x = 1 - y$, y is free, $z = 1$.
- We can read this right from the matrix
- We do say this matrix is in REF and RREF

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2.3: Free variables

- If a column (for a variable) has no pivot, then
  that variable is **free**
- Be careful when reading the answer off the matrix
  $110|1$ means $x + y = 1$, so $x = 1 - y$
- If a variable is free, then (assuming there are any solutions)
  there are **infinitely many solutions**
- What does "no solution" look like in matrix format?

2.3: What if things go wrong?

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- Let's write it out in variables, and see what is going on:

$$x + y = 1 \qquad z = 1 \qquad 0 = 1$$

- Well that is not too bad? $x = 1 - y$, y is free, $z = 1$, and **0=1? What?!**
- **No solution**, **inconsistent**
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Lab time: Activity 2.3b

- Form groups of 1-4 people and finish working on activity 2.3b
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

Homework: Tricky homework type

- Struggling is good; don't worry, don't give up
- People struggle with #1 = #2 = #7 = #8; do #7 first
- People struggle with word problems like #9, #11:
  Read carefully, Do not rush, Check your work,
  You should be able to explain your answer in complete sentences
  You need to learn how to read these to pass MA162 and your major
- People have trouble with diagrams like #5
- I am just waiting to help my students with homework
  Tuesday and Thursday, 2:30pm-4:00pm, CB63
  8 other MA162 instructors also want to help

Quiz: 2.3

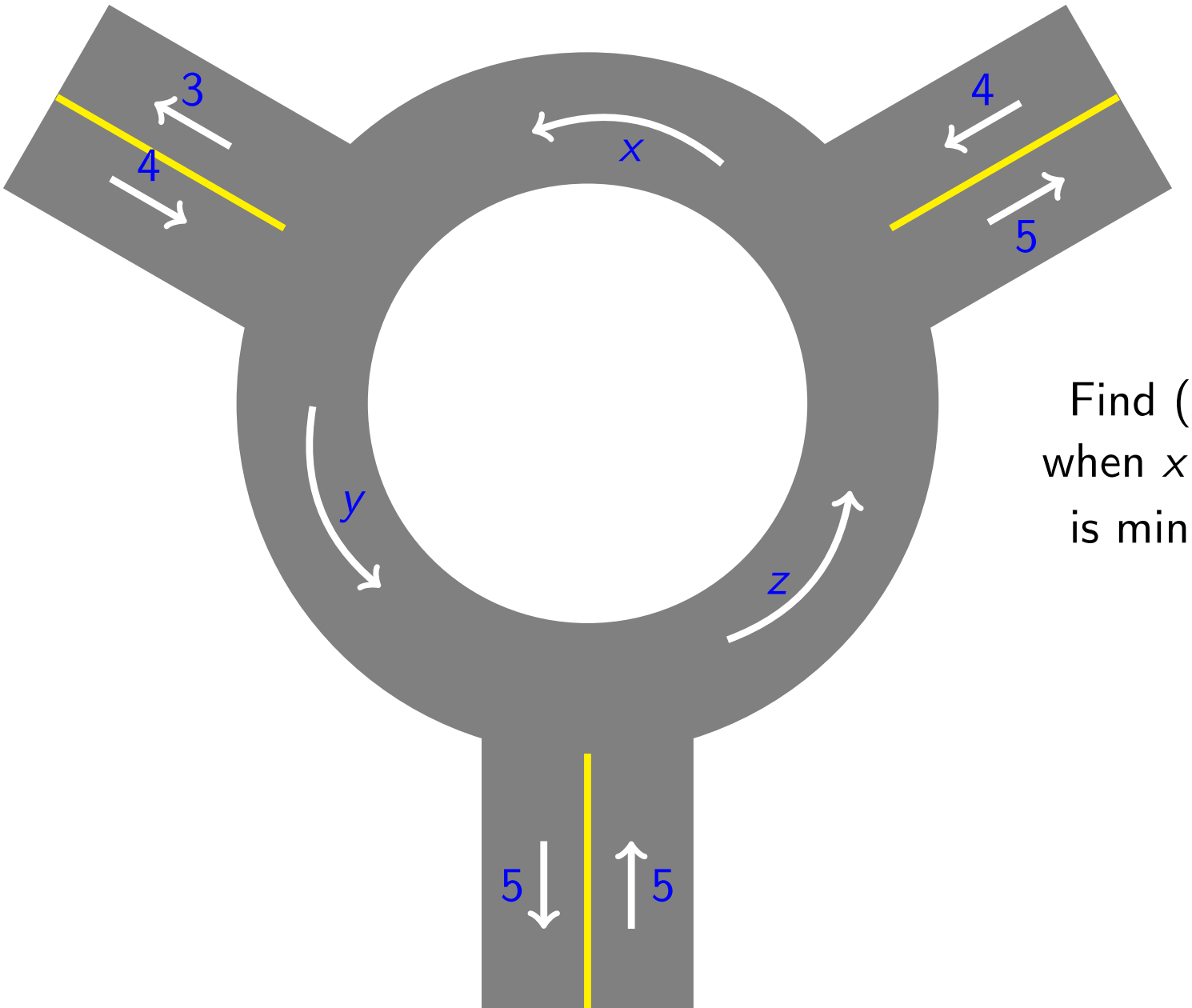
- Bring this matrix to RREF:

$$\left[\begin{array}{ccc|c} 2 & 6 & 6 & 57 \\ 0 & 2 & 0 & 8 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

- Read the answer from this RREF matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Roundabout: Smaller #5



Find (x, y, z)
when $x + y + z$
is minimized.

Line-about: same thing with lines