Examples 2.5: Matrix multiplication

General example of multiplication, rows of the left times columns of the right:

$$\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix} \cdot \begin{bmatrix}7 & 8 & 9 & 10\\ 11 & 12 & 13 & 14\end{bmatrix} = \begin{bmatrix}1\cdot 7+2\cdot 11 & 1\cdot 8+2\cdot 12 & 1\cdot 9+2\cdot 13 & 1\cdot 10+2\cdot 14\\ 3\cdot 7+4\cdot 11 & 3\cdot 8+4\cdot 12 & 3\cdot 9+4\cdot 13 & 3\cdot 10+4\cdot 14\\ 5\cdot 7+6\cdot 11 & 5\cdot 8+6\cdot 12 & 5\cdot 9+6\cdot 13 & 5\cdot 10+6\cdot 14\end{bmatrix}$$

Size must match:

$$\begin{bmatrix}1 & 2 & 47\\ 3 & 4 & 57\\ 5 & 6 & 67\end{bmatrix} \cdot \begin{bmatrix}7 & 8 & 9 & 10\\ 11 & 12 & 13 & 14\end{bmatrix}$$
$$= \begin{bmatrix}1\cdot 7+2\cdot 11+47\cdot ? & 1\cdot 8+2\cdot 12+47\cdot ? & 1\cdot 9+2\cdot 13+47\cdot ? & 1\cdot 10+2\cdot 14+47\cdot ?\\ 3\cdot 7+4\cdot 11+57\cdot ? & 3\cdot 8+4\cdot 12+57\cdot ? & 3\cdot 9+4\cdot 13+57\cdot ? & 3\cdot 10+4\cdot 14+57\cdot ?\\ 5\cdot 7+6\cdot 11+67\cdot ? & 5\cdot 8+6\cdot 12+67\cdot ? & 5\cdot 9+6\cdot 13+67\cdot ? & 5\cdot 10+6\cdot 14+67\cdot ?\end{bmatrix}$$

= **nonsense;undefined**

Some matrices don't change anything (identity matrix):

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}11 & 12 & 13 & \dots\\ 21 & 22 & 23 & \dots\\ 31 & 32 & 33 & \dots\\ 41 & 42 & 43 & \dots\end{bmatrix} = \begin{bmatrix}1\cdot \mathbf{11}+0\cdot 21+0\cdot 31+0\cdot 41 & 12 & 13 & \dots\\ 0\cdot 11+1\cdot \mathbf{21}+0\cdot 31+0\cdot 41 & 22 & 23 & \dots\\ 0\cdot 11+0\cdot 21+1\cdot \mathbf{31}+0\cdot 41 & 32 & 33 & \dots\\ 0\cdot 11+0\cdot 21+0\cdot 31+1\cdot \mathbf{41} & 42 & 43 & \dots\end{bmatrix} = \begin{bmatrix}11 & 12 & 13 & \dots\\ 21 & 22 & 23 & \dots\\ 31 & 32 & 33 & \dots\\ 41 & 42 & 43 & \dots\end{bmatrix}$$

Examples 2.6: Matrix inverses

Sometimes we want to divide matrices: Solve for X in $AX = B$. Use $RREF(A|B) = (I|X)$.
If $A = \left[\begin{smallmatrix}1 & 2\\ 3 & 7\end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]$, then solve for X in $AX = B$.

$$\left[\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 3 & 7 & 0 & 1\end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & 1 & -3 & 1\end{array}\right] \xrightarrow{R_1-2R_1} \left[\begin{array}{cc|cc}1 & 0 & 7 & -2\\ 0 & 1 & -3 & 1\end{array}\right]$$

Check that $X = \left[\begin{smallmatrix}7 & -2\\ -3 & 1\end{smallmatrix}\right]$ works:

$$A\cdot X = \begin{bmatrix}1 & 2\\ 3 & 7\end{bmatrix} \cdot \begin{bmatrix}7 & -2\\ -3 & 1\end{bmatrix} = \begin{bmatrix}1\cdot 7+2\cdot(-3) & 1\cdot(-2)+2\cdot 1\\ 3\cdot 7+7\cdot(-3) & 3\cdot(-2)+7\cdot 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = B$$

If $A = \left[\begin{smallmatrix}1 & 2 & 3\\ 0 & 1 & 4\\ 0 & 0 & 1\end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix}1 & 2 & 3\\ 1 & 1 & 1\\ 2 & 2 & 2\end{smallmatrix}\right]$, then solve for X in $AX = B$.

$$\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 2 & 3\\ 0 & 1 & 4 & 1 & 1 & 1\\ 0 & 0 & 1 & 2 & 2 & 2\end{array}\right] \xrightarrow{R_2-4R_3} \left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 2 & 3\\ 0 & 1 & 0 & -7 & -7 & -7\\ 0 & 0 & 1 & 2 & 2 & 2\end{array}\right] \xrightarrow{R_1-3R_3}$$
$$\left[\begin{array}{ccc|ccc}1 & 2 & 0 & -5 & -4 & -3\\ 0 & 1 & 0 & -7 & -7 & -7\\ 0 & 0 & 1 & 2 & 2 & 2\end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 9 & 10 & 11\\ 0 & 1 & 0 & -7 & -7 & -7\\ 0 & 0 & 1 & 2 & 2 & 2\end{array}\right]$$

Check that $X = \left[\begin{smallmatrix}9 & 10 & 11\\ -7 & -7 & -7\\ 2 & 2 & 2\end{smallmatrix}\right]$ works:

$$A\cdot X = \begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}9 & 10 & 11\\ -7 & -7 & -7\\ 2 & 2 & 2\end{bmatrix} = \begin{bmatrix}1\cdot 9+2\cdot(-7)+3\cdot 2 & 1\cdot 10+2\cdot(-7)+3\cdot 2 & 1\cdot 11+2\cdot(-7)+3\cdot 2\\ 0\cdot 9+1\cdot(-7)+4\cdot 2 & 0\cdot 10+1\cdot(-7)+4\cdot 2 & 0\cdot 11+1\cdot(-7)+4\cdot 2\\ 0\cdot 9+0\cdot(-7)+1\cdot 2 & 0\cdot 10+0\cdot(-7)+1\cdot 2 & 0\cdot 11+1\cdot(-7)+4\cdot 2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 1\\ 2 & 2 & 2\end{bmatrix} = B$$

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Quiz on 2.6 (and 2.5): Matrix division

1. Find the inverse of the matrix

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

2. Solve $AX = B$ for X (and check your work) when

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} \quad & \quad & \quad \\ & & \end{bmatrix}$$

3. Is $A + B$ defined? ______________ Is $A - C$ defined? ______________
   Is $A \cdot B$ defined? ______________ Is $A \cdot C$ defined? ______________

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Activity 2.5: Matrix multiplication

1. What size is AB if A is 2×5 and B is 5×13?

2. What size is $A + B$ if A is 2×5 and B is 5×13?

3. What size is $A + B$ if A is 2×5 and B is 2×5?

4. What size is AB if A is 2×5 and B is 2×5?

5. If $AX = B$ and A is 3×3 and B is 3×7, what size is X?

6. If A has 3 rows and AA is defined, how many columns does A have?

7. If $AA = A^2$ is defined, must $AAA = A^3$ be defined? Explain why or give an example where it is not defined.

8. If AB is defined, must BA be defined? Explain why or give an example where it is not defined.

Activity 2.6: Matrix division

Find the inverse of $A = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ by using row reduction:

$$\left[\ A \mid I\ \right] = \left[\begin{array}{ccc|ccc} 1 & -2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|ccc} & & & & & \\ & & & & & \\ & & & & & \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|ccc} & & & & & \\ & & & & & \\ & & & & & \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|ccc} & & & & & \\ & & & & & \\ & & & & & \end{array}\right] = \left[\ I \mid A^{-1}\ \right]$$

Solve $AX = B$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$ by using row reduction:

$$\left[\ A \mid B\ \right] = \left[\begin{array}{cc|ccc} 1 & 2 & 6 & 5 & 4 \\ 2 & 3 & 9 & 8 & 7 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cc|ccc} & & & & \\ & & & & \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cc|ccc} & & & & \\ & & & & \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cc|ccc} & & & & \\ & & & & \end{array}\right] = \left[\ I \mid X\ \right]$$

Check your work:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} & & \\ & & \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$$