MA162: Finite mathematics

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SCHEDULE:

- Exams are being adjusted (up),
  so expect Friday, Feb 11th, 2010 on mathclass.org.
- HW B1 is due Monday, Feb 22nd, 2010.
- HW B2 is due Monday, Mar 1st, 2010.
- HW B3 is due Sunday, Mar 7th, 2010.
- Exam 2 is Monday, Mar 8th, 5:00pm-7:00pm.
- My office hours are Tuesday and Thursday, 2:30pm-4:00pm in CB63

Today we will cover 2.6 and review 2.5: matrix division

2.5 and 2.6: Overview

- Tuesday we learned to add and subtract matrices of the same size
- We also learned how to multiply a matrix by a number
- And how to multiply two matrices togther
- Today we handle the **size** issue for multiplication
- And how to **divide** matrices

2.4: Review from yesterday

- Quiz answers:

$$\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix} + \begin{bmatrix}7 & -7\\8 & -8\\9 & -9\end{bmatrix} = \begin{bmatrix}1+7 & 2+(-7)\\3+8 & 4+(-8)\\5+9 & 6+(-9)\end{bmatrix} = \begin{bmatrix}8 & -5\\11 & -4\\14 & -3\end{bmatrix}$$

$$\begin{bmatrix}1\\2\end{bmatrix} + 3\cdot\begin{bmatrix}4\\5\end{bmatrix} = \begin{bmatrix}1+3\cdot(4)\\2+3\cdot(5)\end{bmatrix} = \begin{bmatrix}13\\17\end{bmatrix}$$

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\cdot\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix} = \begin{bmatrix}1\cdot(5)+2\cdot(7) & 1\cdot(6)+2\cdot(8)\\3\cdot(5)+4\cdot(7) & 3\cdot(6)+4\cdot(8)\end{bmatrix} = \begin{bmatrix}19 & 22\\43 & 50\end{bmatrix}$$

2.5: Sizes for multiplication

- To multiply $A \cdot B$ we take the rows of A and "multiply" them against the columns of B
- We need each row of A to be the same length as each column of B They need to "match up"
- In other words, to multiply A and B, the number of columns of A must be equal to the number of rows of B
- 3×4 times 4×5 is good
  3×4 times 5×6 is not good, the rows of A have only 4 numbers, but the columns of B have 5
- If A is 3×4 and B is 4×5,
  then each "little multiplication" adds up 4 products

  (Rows $\times$ Columns)

2.5: Size for multiplication

- How big is $A \cdot B$?
- If A is 3×2 and B is 2×4 then $A \cdot B$ is 3×4:
  Each "little multiplication" adds up 2 products, and there are 3 rows of products, and 4 columns

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 7 + 2 \cdot 11 & 1 \cdot 8 + 2 \cdot 12 & 1 \cdot 9 + 2 \cdot 13 & 1 \cdot 10 + 2 \cdot 14 \\ 3 \cdot 7 + 4 \cdot 11 & 3 \cdot 8 + 4 \cdot 12 & 3 \cdot 9 + 4 \cdot 13 & 3 \cdot 10 + 4 \cdot 14 \\ 5 \cdot 7 + 6 \cdot 11 & 5 \cdot 8 + 6 \cdot 12 & 5 \cdot 9 + 6 \cdot 13 & 5 \cdot 10 + 6 \cdot 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 32 & 35 & 38 \\ 65 & 72 & 79 & 86 \\ 101 & 112 & 123 & 134 \end{bmatrix}$$

2.5: An easy multiplication, the identity

- There is a matrix that doesn't change things when it multiplies against them:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 11 & 12 & 13 & \ldots \\ 21 & 22 & 23 & \ldots \\ 31 & 32 & 33 & \ldots \\ 41 & 42 & 43 & \ldots \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 12 & 13 & \ldots \\ 0 \cdot 11 + 1 \cdot 21 + 0 \cdot 31 + 0 \cdot 41 & 22 & 23 & \ldots \\ 0 \cdot 11 + 0 \cdot 21 + 1 \cdot 31 + 0 \cdot 41 & 32 & 33 & \ldots \\ 0 \cdot 11 + 0 \cdot 21 + 0 \cdot 31 + 1 \cdot 41 & 42 & 43 & \ldots \end{bmatrix}$$

- Make sure the size of the matices match though!

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Lab time: Activity 2.4

- Form groups of 1-4 people and finish working on activity 2.4
- You will be given a **short quiz** on the material at the end
- Collaboration is encouraged, but write down your own thoughts
- Write neatly enough for your own notes,
  but you will not hand in anything but the quiz

2.6: Matrix division

- There are several ways to do matrix division, see book for some "cute tricks"
- We'll cover one systematic, basically easy way
- And we **already know it**, we just use RREF:
- If you know A and B, then to solve $AX = B$
  put the augmented matrix $(A|B)$ into RREF as $(I|X)$
- In other words, $RREF(A|B) = (I|X)$
- **inverses** are solving $AX = I$, $X = A^{-1}$, so we use RREF there too

2.6: Why is the inverse useful?

- The inverse allows you to solve $AX = B$ using matrix multiplication instead of RREF
- $A^{-1}A = I$
- $A^{-1}AX = IX = X$
- If $AX = B$, then multiply both sides on the left by A^{-1}
  then $A^{-1}AX = A^{-1}B$
  so $X = A^{-1}B$
- Multiply by the inverse does the same thing as the long RREF
- Of course to find the inverse, we use RREF

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Homework: Tricky homework type

- Struggling is good; don't worry, don't give up
- You should now be able to (struggle to) do all HW B1
  Today we learned to do #9, #3, #7, #5 (in order of easy)
- Some of the problems are easy; you can do them today
  (see Tuesday's lab)
- I am just waiting to help my students with homework
  Tuesday and Thursday, 2:30pm-4:00pm, CB63
  8 other MA162 instructors also want to help

Quiz: 2.5 and 2.6

- Find the inverse of the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

- Solve $AX = B$ for X (and check your work) when $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

- Is $A + B$ defined? Is $A - C$ defined? Is $A \cdot B$ defined? Is $A \cdot C$ defined?