Examples 3.1: Graphing inequalities

To **graph a line**, you only need two points on the line. For equations like $3x + 4y = 12$, I like to plug-in $x = 0$ to get one point: $0 + 4y = 12$ so $(x = 0, y = 3)$ is a point on the line; then plug-in $y = 0$ to get another: $3x + 0 = 12$ so $(x = 4, y = 0)$ is a point on the line.

To **find the equation of a line from the graph** you only need two points. If the points are $(1, 2)$ and $(31, 302)$, then the slope is $(302 - 2)/(31 - 1) = 300/30 = 10$. Most people then like to use $y = mx + b$: $302 = 10(31) + b$, so $302 = 310 + b$, so $b = -8$. The line has equation $y = 10x - 8$. Every point on the line looks like $(x, 10x - 8)$.

To find **where two lines intersect**, one "sets them equal". For instance to find the intersection of $y = 3x + 5$ and $y = 7x + 9$ one solves $3x + 5 = 7x + 9$ as $-4 = 4x$, $x = -1$, and then plugs x into the equations $y = 3(-1) + 5 = 2$ or $y = 7(-1) + 9 = 2$. The intersection is $(x = -1, y = 2)$.

To decide **which side of a line** is the correct side, just plug in a point and check. Which side of $4x + 7y \leq 10$ do we want? Try $(x = 0, y = 0)$ as $0 + 0 \leq 10$. That is the correct side, so that is the side we want ("below the line").

Finding **corners** without graphing can be tricky. A corner has **two** properties: it is the intersection of two of the lines from the inequalities, **and** it satisfies all of the other inequalities too. For instance $x \geq 2, x + y \leq 3, x \geq 0, y \geq 0$ has the two lines $x = 0$ and $y = 0$ whose intersection is the origin $(x = 0, y = 0)$. However, this is **not** a corner, because it does not satisfy $x \geq 2$.

Examples 3.2: Word problems

You must read word problems and write down a mathematical model of the problem. In general, that means deciding which things are variables, what equalities and inequalities relate those variables, and which variables should be solved for. In this chapter, this means finding the **choices** (the variables), the **constraints** (the inequalities), and the **objective function** (the goal, usually profit to be maximized).

Given: Roger has \$3000 to invest and is considering three investment options: a savings account earning 0.1% interest, a certificate of deposit earning 1% interest, or a mutual fund which could earn 3% interest, but might also default (lose all his money). He needs to have at least \$1000 available at all times to cover emergencies (so does not want to invest it in the CD or the mutual fund), and at the end of the year he needs to have at least \$2000 in order cover his upcoming (used) car purchase, so he does not want to risk putting that money in the unsecured mutual fund.

Model: Let S, C, and M denote the number of dollars Roger invests in the savings, certificate of deposit, and mutual fund respectively. The constraints are that $S, C, M \geq 0$, $S + C + M \leq 3000$, $C + M \leq 2000$, $M \leq 1000$. The objective function is to maximize his (expected) end of year value: $1.001A + 1.01B + 1.03C$.

Bonus: Try to find the optimal solution! I got \$41 in interest.

Given: You are assisting a farmer with planning for the next year. He has 150 acres of land for crops, \$7400 in capital for seed, fertilizer, water, etc., and 3300 hours of labor available to work the fields. He is considering two types of crops. The first costs \$40 and 20 hours of labor per acre, and the second costs \$60 and 25 hours of labor per acre. You expect the crops to produce about \$150 and \$200 per acre in revenue, respectively.

Model: Let A and B be the number of acres the farmer allocates to the two crops. Then the constraints are $A + B \leq 150$, $40A + 60B \leq 7400$, $20A + 25B \leq 3300$, and $A \geq 0, B \geq 0$. The objective is to maximize the revenue $150A + 200B$.

Bonus: Try to find the optimal solution! I got \$25,750 in revenue.

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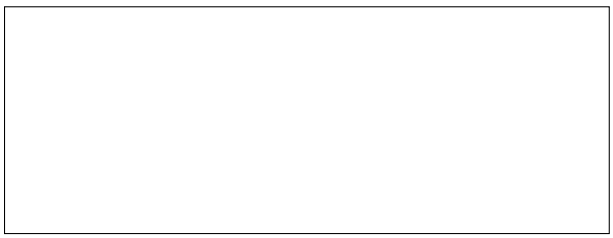
Quiz on 3.2: Word problems

Sample problem: A mining company operates two mines for the purpose of extracting gold and silver. The Saddle Mine costs $14,000 a day to operate and it yields 50 oz of gold and 3000 oz of silver each day. The Horseshoe mine costs $16,000 a day to operate and it yields 75 oz of gold and 1000 oz of silver each day. Company management has set a target of at least 650 oz of gold and 18,000 oz of silver. How many days should each mine be operated so that the target can be met at minimum cost?

Quiz: Setup the model for this word problem. Do not solve it.

Your answer: Name and describe the variables:



List the constraints:

- _____
- _____
- Standard inequalities: _____

What is the objective function (goal)? _____

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Activity 3.2a: Spending your time

Despite the hard economic times you find that you have two job openings (part time): customer service and floor detail at a local shop, and data entry at a check clearing house. The local shop pays $7.50/hr and data entry pays $10.00/hr. You know if you drop all your extra-curricular activities you can still maintain your grades if you avoid working more than 20 hours/week. However, you also know you are prone to eye-strain if you stare at a computer more than 15 hours/week at work. The first job has you staring at a computer 40% of the time, while the second job is nothing but staring at a computer. You also suffer from a serious texting addiction, and don't want to spend more than 10 hours away from your friends. At the customer service job, it looks like you'll be isolated and out of touch (aka talking to customers) 60% of the time, while at the data entry job you figure you can stay in touch 80% of the time. How much should you work to maximize your earnings while staying true to your school commitments, your doctor's orders, and your friends?

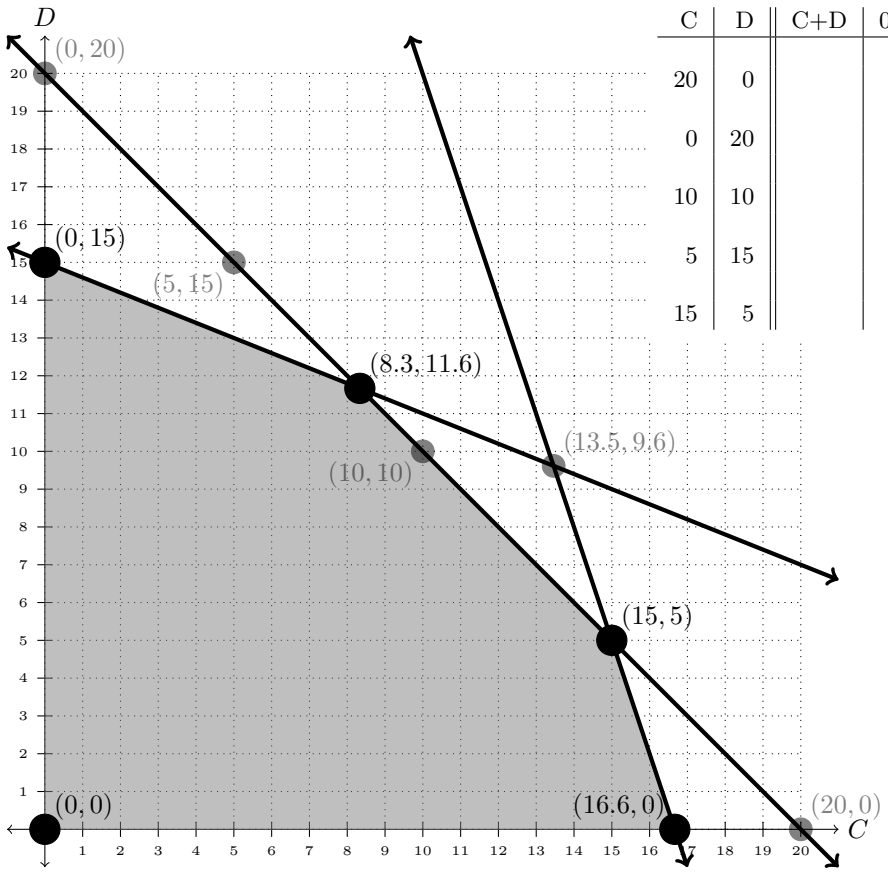
Name and describe the variables:

List the constraints:

- _____
- _____
- _____
- Standard inequalities: _____

What is the objective function (goal)? _____

Activity 3.2b: Finding good feasible solutions



C	D	C+D	0.4C + D	0.6C +0.2D	7.5C + 10D
20	0				
0	20				
10	10				
5	15				
15	5				

- Let C be the number of hours worked at the local shop,
- Let D be the number of hours worked at data entry. Then, subject to:
- $C + D \leq 20$ (academic constraint)
- $0.4C + D \leq 15$ (medical constraint)
- $0.6C + 0.2D \leq 10$ (social constraint)
- Maximize $7.5C + 10D$ (earnings)

Fill out the table above to answer these questions:

How much do you make working all 20 hours at the local shop? How many hours do you spend in front of the computer at work? How many hours do you spend isolated from your friends? Is this feasible?

How much do you make working all 20 hours at data entry? How many hours do you spend in front of the computer at work? How many hours do you spend isolated from your friends? Is this feasible?

Consider a mixed strategy where you spend 10 hours at both. How much do you make? How many hours do you spend in front of the computer at work? How many hours do you spend isolated from your friends? Is this feasible?

What about 5 hours at the local shop and 15 at data entry? How much do you make? How many hours do you spend in front of the computer at work? How many hours do you spend isolated from your friends? Is this feasible?

What about 15 hours at the local shop and 5 at data entry? How much do you make? How many hours do you spend in front of the computer at work? How many hours do you spend isolated from your friends? Is this feasible?

What about spending 13.5 hours at the local shop and 9.6 at data entry? Is this feasible? Is it a corner? Why would anyone consider this?

What is your advice? How many hours at the local shop, how many hours at data entry? How much do they make? Don't violate any of the constraints! Who cares how much money you earn, if you have to drop out of school, because you went blind, and lost all your friends!