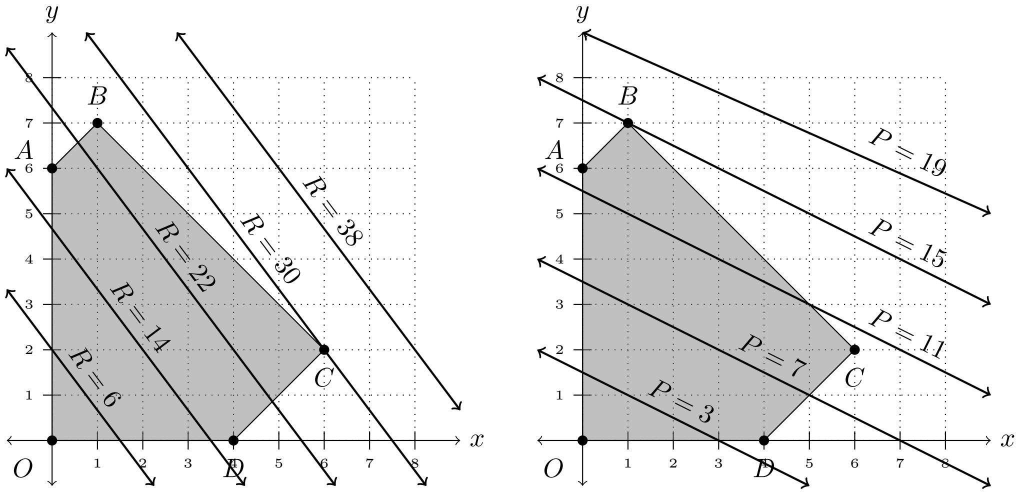
Examples 3.3: Method of corners



The region from the example sheet 3.1 was given by the inequalities $x \geq 0$, $y \leq x+6$, $x+y \leq 8$, $x \leq y+4$, $y \geq 0$, and has corners $O = (0,0)$, $A = (0,6)$, $B = (1,7)$, $C = (6,2)$, $D = (4,0)$.

If x represents the number of Aurora bracelets made, and y represents the number of Babylon bracelets made, then we might be operating under the constraints that both Aurora and Babylon require a foot of string, of which we only have 8 feet, and that the friends of Aurora and friends of Babylon have demanded that we make roughly equal numbers, no more than six more Auroras than Babylons, and no more than four more Babylons than Auroras. This would give the above feasible region. Suppose it costs \$3 per Aurora, and \$1 per Babylon to make the bracelets, and we can sell them for \$4 per Aurora, and \$3 per Babylon. How do we maximize our revenue, $R = 4x + 3y$? How do we maximize our profit, $P = x + 2y$?

The left graph shows the "level curves" or "iso-revenue lines" for various revenue levels. For instance the places where we get \$6 in revenue are exactly the points (x, y) where $4x + 3y = 6$, including $(0, 2)$. We can graph the curves for $R = \$6$, $R = \$14$, $R = \$22$, $R = \$30$, and $R = \$38$. Notice each of the "curves" is a line, and each of the lines is parallel. Notice that $R = \$38$ is simply not feasible; it never touches the feasible region; we cannot bring in \$38 under the current constraints. The maximum feasible revenue is \$30, and it occurs at the corner C.

The right graph shows the level curves or "iso-profit lines" for various profit levels. For instance the places where we get \$3 in profit are exactly the points (x, y) where $x + 2y = 3$, including the point $(3, 0)$. We can graph the curves for $P = \$3$, $P = \$7$, $P = \$11$, $P = \$15$, and $P = \$19$. Notice each of the curves is a line, and each of the lines is parallel. Notice that $P = \$19$ is simply not feasible; it never touches the feasible region; we cannot profit \$19 under the current constraints. Notice that $P = 11$ is feasible, but it is not optimal; we can do better and stay feasible. The maximum feasible profit is \$15, and it occurs at the corner B.

Method of corners: To find the optimal value of the objective function on a **bounded** region, just evaluate the objective function on each corner and take the optimal value from that list. If the region is unbounded, you need to be more careful and make sure the objective function does not just keep getting better as you get farther and farther off the page.

Example: Revenue We evaluate the revenue at the 5 corners and take the maximum:

	x	y	R
O	0	0	0
A	0	6	18
B	1	7	25
C	6	2	**30**
D	4	0	16

The maximum $R = 30$ occurs at $C = (6, 2)$.

Example: Profit We evaluate the profit at the 5 corners and take the maximum:

	x	y	P
O	0	0	0
A	0	6	12
B	1	7	**15**
C	6	2	10
D	4	0	4

The maximum $P = 15$ occurs at $C = (1, 7)$.

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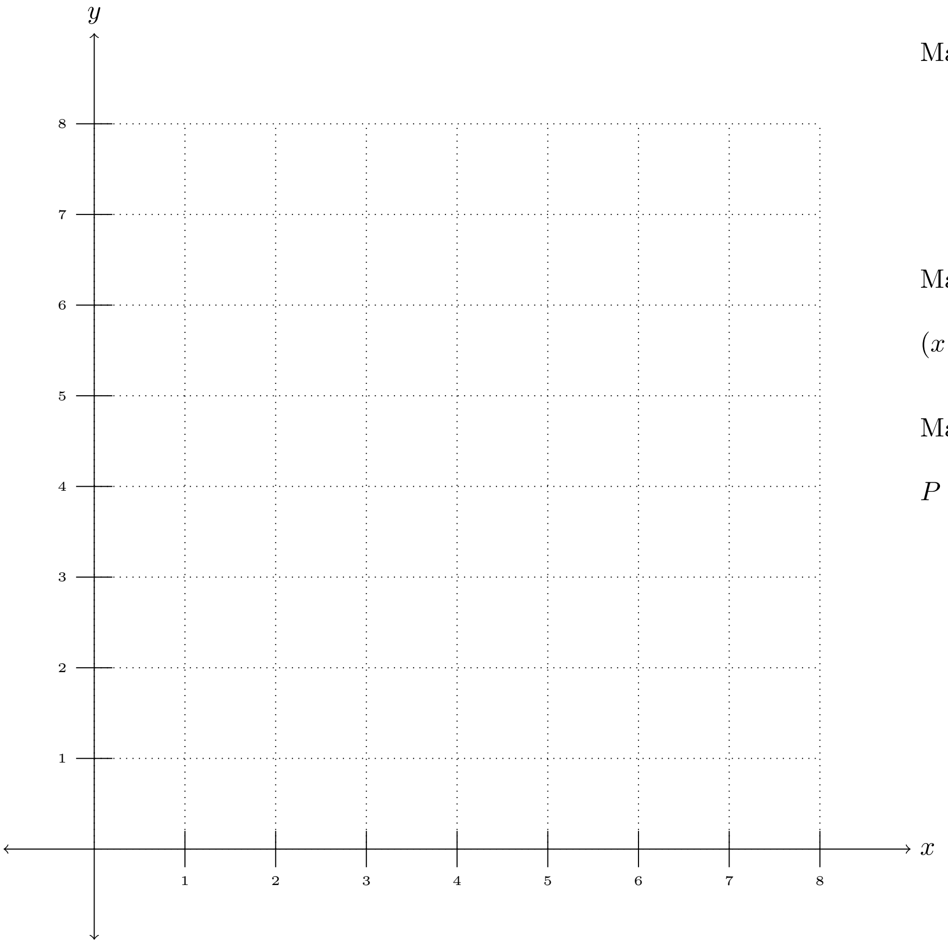
Quiz on 3.3: Method of corners



Maixmize $P = 2x + 3y$ subject to:

- $x + y \leq 6$
- $x \leq 3$
- $x \geq 0$
- $y \geq 0$

Maximum occurs at:

($x =$ _______, $y =$ _______)

Maximum profit is:

$P =$ ______________

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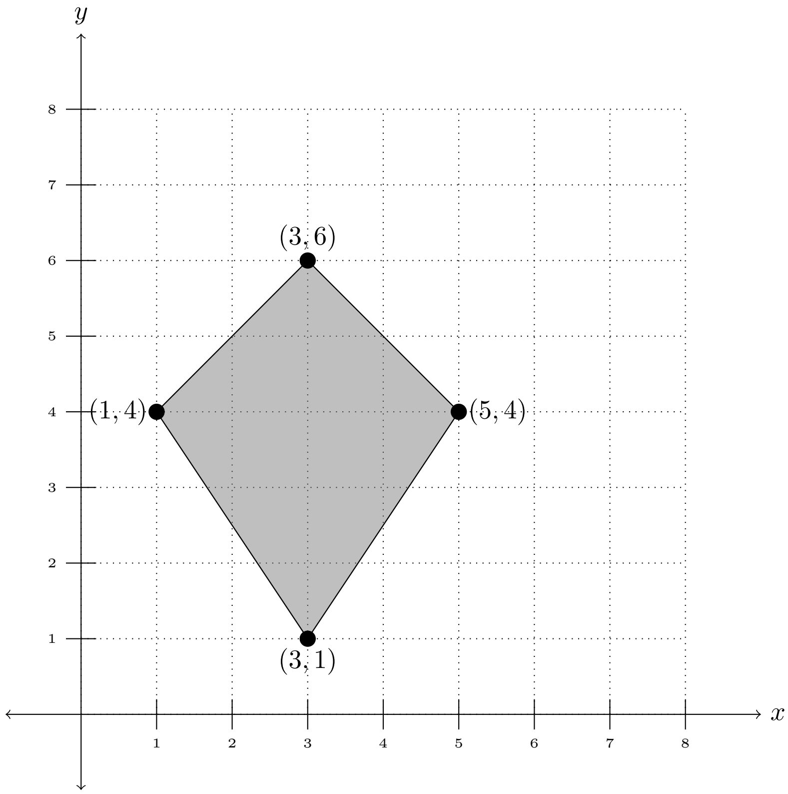
Activity 3.3a: Method of corners



Maximize $P = 3x + 2y$ subject to

- $y \leq x + 3$,
- $x + y \leq 9$,
- $3x - 2y \leq 7$,
- $3x + 2y \leq 11$.

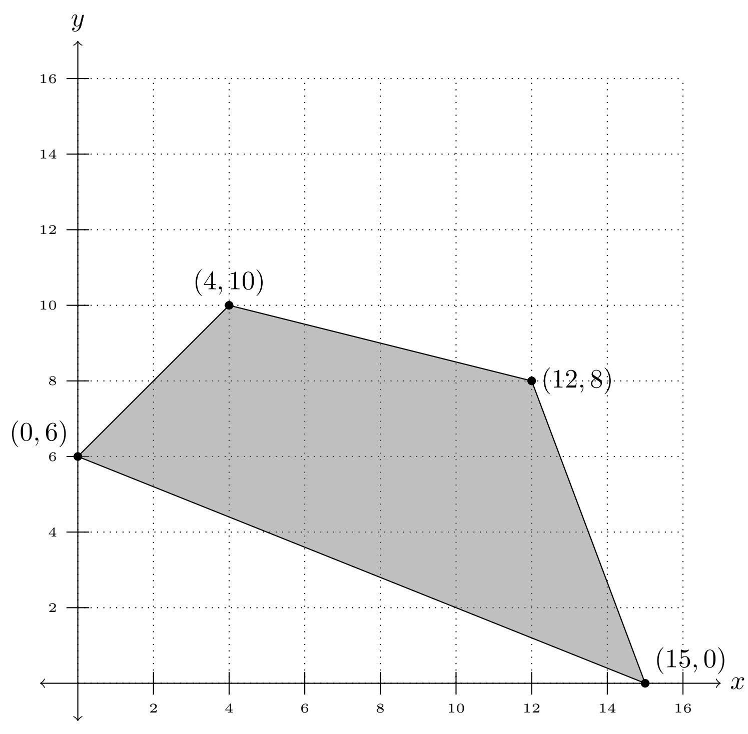
x	y	P
1	4	
3	1	
5	4	
3	6	

Max occurs at:

($x =$ ______, $y =$ ______)

Max profit is:

$P =$ __________



Maximize $P = x + 4y$ subject to

- $y \leq x + 6$,
- $x + 4y \leq 44$,
- $8x + 3y \leq 120$,
- $2x + 5y \leq 30$.

x	y	P
0	6	
15	0	
12	8	
4	10	

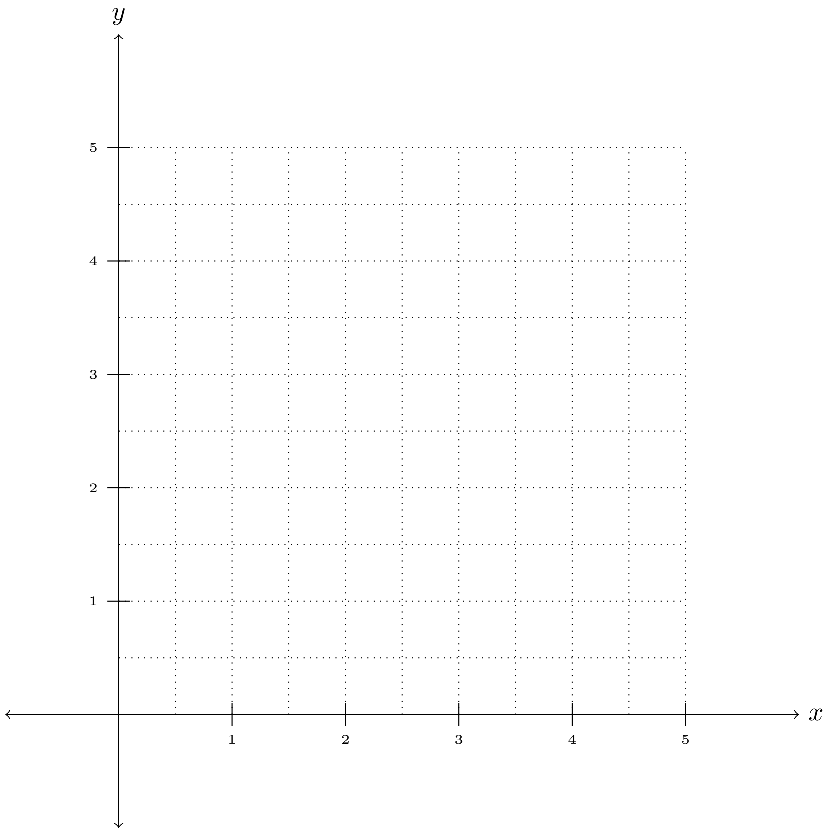
Max occurs at:

($x =$ ______, $y =$ ______)

Max profit is:

$P =$ __________

Activity 3.3b: Method of corners (degeneracy)



Maximize $P = x + 2y$ subject to

- $x + y \leq 4$,
- $2x + y \leq 5$,
- $x \geq 0$, and
- $y \geq 0$.

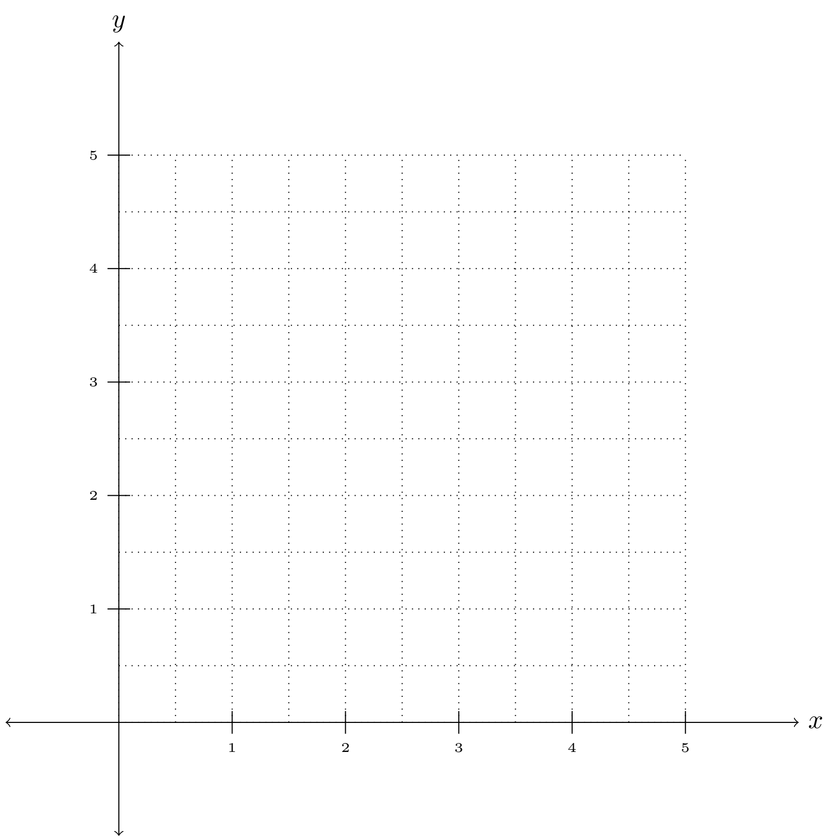
x	y	P

Max occurs at:

$(x =$ ________, $y =$ ________$)$

Max profit is:

$P =$ ______________



Maximize $P = 2x + y$ subject to

- $x + y \leq 4$,
- $2x + y \leq 5$,
- $x \geq 0$, and
- $y \geq 0$.

x	y	P

Max occurs at:

$(x =$ ________, $y =$ ________$)$

Max profit is:

$P =$ ______________