Exam 2 review

2010-03-04

1. Multiply $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{pmatrix}.$$

2. Add $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{pmatrix}$

$$= \begin{pmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{pmatrix}.$$

3. Invert $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$, so that $A^{-1} = \begin{pmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{pmatrix}$.

4. McGruggin's Manufacturing produces farm gates and cattle guards in a time when everyone wants farm gates and cattle guards. He has more customers than he can handle, and he is having trouble running his factory to its full potential. The two products both require time on the welding line and the finishing machine. McGruggin realizes a \$20 profit on farm gates and a \$200 profit on cattle guards. The farm gates require 5 minutes on the welding line and 10 minutes on the finishing machine. The cattle guards require 60 minutes on the welding line, but only 30 minutes on the finishing machine. If McGruggin runs both machines 12 hours a day, how many farm gates and how many cattle guards should he order to be made each day in order to maximize his profit?

Variables: ____

Inequalities: ____

Objective: ____

5. From the initial tableau, one step of the simplex algorithm arrives at the following intermediate tableau:

f	c	w	m	p	RHS	(Ratio)
5	60	1	0	0	720	(144)
10	30	0	1	0	720	(72)
−20	−200	0	0	1	0	

$\xrightarrow{R_1 - \frac{1}{2}R_2;\, R_3 + 2R_2;\, \frac{1}{10}R_2}$

f	c	w	m	p	RHS	(Ratio)
0	45	1	−1/2	0	360	___
1	3	0	1/10	0	72	___
0	−140	0	2	1	1440	

The first step of the simplex algorithm decided to make all the farm gates it could, $f = 72$, but still left 6 hours free for welding, $w = 360$.

Circle the pivot column, calculate the ratios, circle the pivot row, and then apply the three row operations to write down the final tableau:

$\longrightarrow$

f	c	w	m	p	RHS
___	___	___	___	___	___
___	___	___	___	___	___
___	___	___	___	___	___

6. From the final tableau read the solution:

f	c	w	m	p	RHS
0	1	$1/45$	$-1/90$	0	8
1	0	$-1/15$	$2/15$	0	48
0	0	$28/9$	$4/9$	1	2560

Decision is ($f =$ _______, $c =$ _______), resulting in $P =$ _______, and slacks ($w =$ _______, $m =$ _______).

7. **Background:** McGruggin's success has inspired his neighbor McFlugal. McFlugal wants to rent time on McGruggin's welding line and finishing machine, but knows that McGruggin would never rent his assets at a loss. McFlugal lets w denote the price per minute he'll offer to McGruggin for time on the welding line, and m denote the the price per minute he'll offer to McGruggin for time on the finishing machine. If McFlugal too needs 12 hours a day on those machines, his cost would be $C = 720w + 720m$. Now McFlugal knows McGruggin can earn \$20 profit using 5 minutes of welding time and 10 minutes of machine time, so McFlugal is going to have to offer at least enough so that $5w + 10m \geq 20$. Similarly, McGruggin can earn \$200 profit using 60 minutes of welding time and 30 minutes of machine time, so McFlugal is constrained by $60w + 30m \geq 200$. How much should McFlugal offer per minute of welding time and machine time in order to minimize his cost, while still presenting McGruggin with a sane choice?

Simple version: Minimize $C = 720w + 720m$ subject to $5w + 10m \geq 20$, $60w + 30m \geq 200$, $w \geq 0$, $m \geq 0$.

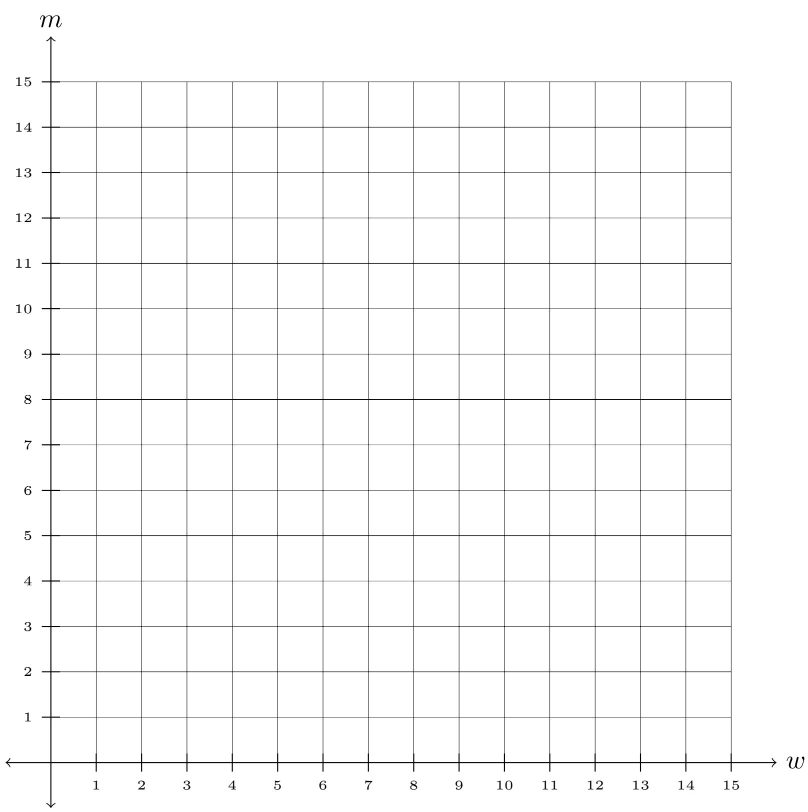
Once you dualize the problem, you get exactly the same tableau as before (though now you think of it as a "dual" tableau), and as you work out the simplex algorithm, everything goes the same so you get the same final tableau. Now read the answer of the primal problem from this final "dual" tableau.

Decision: ($w =$ _______, $m =$ _______), resulting in $C =$ _______ and slacks ($f =$ _______, $c =$ _______)

Interpretation: If McFlugal's offer is accepted, McGruggin will make the same amount of money each day, but will be making $f = 0$ farm gates and $c = 0$ cattle guards himself. Instead all of his money will be coming from the rent. If McGruggin rejects the offer, instead McGruggin will be making $f = 48$ farm gates and $c = 8$ cattle guards, but earning rent on $w = 0$ minutes of welding time and $m = 0$ minutes of machine time.

8. Instead of using the simplex algorithm, use the graphical method:

Simple version: Minimize $C = 720w + 720m$ subject to $5w + 10m \geq 20$, $60w + 30m \geq 200$, $w \geq 0$, $m \geq 0$. Sketch and shade the region defined by the inequalities and use the method of corners to find the minimum value. Be sure to indicate why the minimum occurs at a corner.



w	m	C

Min $C =$ _______________

Occurs at ($w =$ _______, $m =$ _______)