MA162: Finite mathematics

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March 23, 2010

Schedule:

- HW C1 is due Monday, Mar 29th, 2010.
- HW C2 is due Monday, Apr 5th, 2010.
- HW C3 is due Sunday, Apr 11th, 2010.
- Exam 3 is Monday, Apr 12th, 5:00pm-7:00pm.
- **All** alternate exam takers must signup (on mathclass.org) by April 6th.

Today we will cover 5.3: amortized loans. We will be using calculators today.

5.3: Paying off debts

- If you owe \$1000 at 12% interest compounded monthly
  and pay back \$20 per month, how long does it take to pay it off?
- After one month, you owe \$1000 + \$10 interest - \$20 payment, a total of \$990
- So each month the debt goes down by a net \$10?
  Should take 99 more months, or a little more than 8 years.
- After two months, you owe \$990 + \$9.90 interest - \$20 payment, a total of \$979.90
- Now it went down by \$10.10!
  Should take \$979.90/\$10.10 $\approx$ 97 months
  After one month of paying, we estimate two months fewer
  How many is it really?

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5.3: Sinking funds

- Paying off your debt piece by piece is confusing
- Instead, we'll put the money in an equivalent (imaginary!) savings account earning 12% interest compounded monthly
- We put in \$20 per month until we have enough to pay off the debt
- How long will **that** take?
- Setup your variables. What is the decision to be made?
- Let n be the number of months to pay into the account
- You owe $\$1000 \cdot 1.01^n$ at that point
  You have saved $\$20 \cdot (1.01^n - 1)/0.01 = \$2000 \cdot (1.01^n - 1)$

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5.3: Solving the equation

- So set the amount owed to the amount saved:

$$\$1000 \cdot 1.01^n = \$2000 \cdot (1.01^n - 1)$$

- Simplify:

$$1.01^n = 2 \cdot 1.01^n - 2$$

- Simplify further:

$$2 = 1.01^n$$

- Use logarithms:

$$\log(2) = n \log(1.01) \qquad n = \log(2)/\log(1.01) \approx 70$$

- Not 100 months as predicted at first, only 70 (less than 6 years)

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5.3: Changing the payment size

- Maybe six years is still too long, what if we double the payments?
- \$40 per month after n months is worth

$$\$40 \cdot (1.01^n - 1)/0.01 = \$4000 \cdot (1.01^n - 1)$$

- The debt is still $\$1000 \cdot 1.01^n$, and as before:

$$1.01^n = 4 \cdot 1.01^n - 4$$

$$4 = 3 \cdot 1.01^n \qquad 4/3 = 1.01^n$$

$$\log(4/3) = n \cdot \log(1.01) \qquad n = \log(4/3)/\log(1.01)$$

- $n \approx 29$ months, less than three years
- Twice as big payment, half as long?

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5.3: Changing the payment again

- Maybe six years isn't so long; why not twelve?
- Let's cut our payment in half instead, only \$10 per month.
- How long does it take to pay it off?
- Set it up as before:

$$\$1000 \cdot (1.01^n) = \$10 \cdot (1.01^n - 1)/.01$$

$$\$1000 \cdot (1.01^n) = \$1000 \cdot (1.01^n - 1)$$

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- What is n?
- What is going on?

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5.3: Choosing the payment size

- \$10 takes forever, \$20 takes less than six years, \$40 takes less than three years
- What if we want to pay it off in exactly five years?
- After 60 months, the debt is $\$1000 \cdot (1.01^{60}) = \$1816.70$
- After 60 months, the savings is worth $R \cdot (1.01^{60} - 1)/.01 = 81.780R$
- These are equal when $R = \$1816.70/81.780 \approx \$22.24$

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5.3: The superposition principle

- Why do we get the same answer if we
  - Pay into the loan each month, versus
  - Save money into an account with the same interest rate each month, and pay it all at the end
- Suppose you save \$10 every month for a year at 1% interest compounded monthly. How much is the account worth at the end of the year?
- Suppose six months in, you decide to save \$20 every month instead. How much is the account worth at the end of the year?

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5.3: Summary

- Today we learned about **sinking funds** and **amortized loans**

- You are now ready to complete HWC1
  and half of HWC2 (#s 1,2,3,5)

- Make sure to take advantage of office hours