MA162: Finite mathematics

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March 30, 2010

SCHEDULE:

- HW C1 is due Wednesday, Mar 31st, 2010.
- HW C2 is due Monday, Apr 5th, 2010.
- HW C3 is due Sunday, Apr 11th, 2010.
- Exam 3 is Monday, Apr 12th, 5:00pm-7:00pm.
- **All** alternate exam takers must signup (on mathclass.org) by April 6th.

Today we will cover 6.2: counting.

6.1: Quiz review

1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$, then
$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cap B = \{2, 4\}$
$A - B = \{1, 3, 5\}$
$B - A = \{6\}$

2. $X \cup Y = Y \cup X$
$X \cap Y = Y \cap X$
$A - B \neq B - A$
$X - X = X - X$
$A - B \neq A \cap B$
If $X = \{\}$ has nothing in it, then $X - Y = X = X \cap Y$, but no other time.

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- If x took either cream or sugar or both, then $70 \leq x \leq 100$ and the number that took both is $60 + (70 - x)$, since each one beyond 70 means a sugar taker who didn't take cream.

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- Suppose we know that there were 200 people in the testing pool. About how many were drug users?
- Assuming exactly 5% of non-users returned positive, there is a unique answer. Let me know when you've found it.

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- $10 = x + (200 - x) \cdot (5\%) = x + 10 - x/20$ so $0 = \frac{19}{20}x$, $x = 0$
- All 10 are false positives; 100% wrong, but 95% accurate?
  Be careful what you are counting.

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- Then $n(C)$, the number of people in C, is 70 and $n(S) = 60$.

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- Let's redo the coffee survey:
- Let C be the set of people who take cream, and S be the set of people who take sugar
- Then $n(C)$, the number of people in C, is 70 and $n(S) = 60$.
- We want $n(C \cap S)$, but we noticed it depended on $x = n(C \cup S)$:

$$n(C) + n(S) = n(C \cap S) + n(C \cup S)$$

$$70 + 60 = n(C \cap S) + x$$

$$n(C \cap S) = 130 - x$$

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- Well, what if those were exactly the 20 people that didn't eat dinner? Could be 0 that got three square meals, but it'd be a heck of a coincidence.

- What more do we need to know? The number that ate both breakfast and lunch is important, right? If it was any bigger than 20, then we'd have some three square meals people.

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- Just like before, there is a formula relating all of these things:

  $$n(B)+n(L)+n(D)+n(B\cap L\cap D) = n(B\cup L\cup D)+n(B\cap L)+n(L\cap D)+n(D\cap B)$$

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- We plugin to get:

$$55 + 65 + 80 + n(B \cap L \cap D) = 100 + 34 + 46 + 40$$

$$n(B \cap L \cap D) = 100 + 34 + 46 + 40 - 55 - 65 - 80 = 20$$

6.2: Summary

- We learned the notation $n(A) =$ the number of things in the set A

- We learned the basic inclusion-exclusion formulas:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

  and

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

- Make sure to complete HWC1 and HWC2