MA162: Finite mathematics

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April 6, 2010

Schedule:

- HW C3 is due Sunday, Apr 11th, 2010.
- Exam 3 is Monday, Apr 12th, 5:00pm-7:00pm.

• All alternate exam takers must signup (on mathclass.org) by today. Today we will cover 6.4: permutations. (1a) 4 shoes, 6 tops, (2+3) bottoms; so 4*6*5 = 120 outfits

(1b) +2 tops, -1 shoes; 3*8*5 = 120 outfits

(2a) 20 choices for first, each has 19 choices left for second, each has 18 choices left for third; 20*19*18=6840 possible trifectas

(2b) -2 horses; 18*17*16=4896 possible trifectas

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- So eight for the second part, and six for the third; 10*8*6 = 480 ways.

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• 4*2 ways counting order, then divide by two to ignore order

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- 8! ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCkY.

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- Then 5 ways of choosing the goalie.
- Total is: (1365)(165)(56)(5) ways of choosing the first string

• We learned to handle symmetries in our counting, especially **permutations**, and **combinations**.

Make sure to complete HWC3 ASAP, and begin work on the practice exam

• Be ready to discuss the practice exam next class; bring a copy