MA162: Finite mathematics

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April 15, 2010

Schedule:

- HW D1 is due Monday, Apr 19th, 2010.
- HW D2 is due Monday, Apr 26th, 2010.
- HW D3 is due Friday, Apr 30th, 2010.
- Final Exam is Thursday, May 6th, 6:00pm-8:00pm
- There is an alternate signup sheet (for Friday, May 7th)

Today we will cover 7.2: Probability

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- The event "rolling saved us money" is all those pairs that total to more than 6.
- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- Explicitly:

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- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?
- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- Well, worst case scenario is 100 bulbs break every day all week, so we could keep 700 bulbs in stock.
- However, that's not very likely to happen and quite expensive to plan for.
- If each bulb is independent, that is $(0.1\%)^{700}\approx 0\%$ chance of this happening

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- Total is: 0.844 = 84.4% chance that at most one breaks, so not too bad. Every 6 weeks you'll have a light out and no replacement, but not too bad.

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- What are the odds that 10 is enough?
- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$, exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$, exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2 (99.9\%)^{6998} \approx 2.2\%$, ...

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• Total is: 0.902 = 90.2% chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

Bigger is better

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- This is why insurance is important; the risk to one person is great
- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

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- 12 bad out of 30 total is 40% chance for showers (of fists)