MA162: Finite mathematics

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April 22, 2010

Schedule:

- HW D2 is due Monday, Apr 26th, 2010.
- HW D3 is due Friday, Apr 30th, 2010.
- Final Exam is Thursday, May 6th, 6:00pm-8:00pm
- There is an alternate signup sheet due Today
- Practice final is available now

Today we will cover 7.4: Counting as probability

• P(burger) = 60%, P(breakfast) = 20%, $P(burger \cup breakfast) = 70\%$

So P(burger \cap breakfast) = 60% + 20% - 70% = 10% overlap

•
$$P(<2) = 44\%$$
, $P(<3) = 58\%$, $P(\geq 4) = 10\%$

$$P({2,3}) = P(<4) - P(<2) = (100\% - 10\%) - 44\% = 46\%$$

P(2) = P(< 3) - P(< 2) = 14%

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- Out of 4 people, at least 2 have the same sign?

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- Out of 3 people, at least 2 have the same sign?
- Out of 4 people, at least 2 have the same sign?
- Out of 5 people, at least 2 have the same sign?

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- Or is it? Sample space is (5)(4)/2 = 10 unordered draws Event is 1 good draw Probability is $\frac{1}{10} = 10\%$
- If you had a deck of 52 cards and you drew 5 cards, would it matter if you drew them one by one or five at a time?

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 - A pair is not drawn
 - Two black cards are drawn
 - Two cards of the same suit are drawn

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 - You tie? That is, the other guy gets a pair of 7s

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- Can we view this as a counting problem?
- Yes, rewrite the base sample space as 100 people with *N* an **event** with 42 people in it. Now we can treat it just like a card or dice problem.

7.4: Coffee versus tea

• In a group of 110 people, 52 drink coffee, 41 drink tea, 39 drink neither. What is the probability that a random person:

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- If you knew they drank coffee, what is the probability they also drank tea?