MA162: Finite mathematics

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April 27, 2010

Schedule:

- HW D3 is due Friday, Apr 30th, 2010.
- Final Exam is Thursday, May 6th, 6:00pm-8:00pm
- **Practice final** is available now, bring it to class Thursday

Today we will cover 7.5: Conditional probabilities

7.4 Quiz Review

- 5 people sit in a circle, how likely is it that 2 sit together?

  1st can sit anywhere, 2nd can sit in 2 out of 4 seats, so $2/4 = 50\%$

- Chance that 7 choices from 7 things have at least 2 equal?

  7^7 possible, if not, then all different, so 7! wrong ways, so $1 - 7!/7^7 = 116929/117649 \approx 99.4\%$

- 200 total, 10 bad, choose 5, how likely all are good?

  $C(200, 5)$ possible, $C(190, 5)$ are all good, so $C(190, 5)/C(200, 5) = \frac{190}{200}\frac{189}{199}\frac{188}{198}\frac{187}{197}\frac{186}{196} \approx 77.2\%$

- 195 total, 10 bad, choose 5, how likely all are good?

  $C(195, 5)$ possible, $C(185, 5)$ are all good, so $C(185, 5)/C(195, 5) = \frac{185}{295}\frac{184}{194}\frac{183}{193}\frac{182}{192}\frac{181}{191} \approx 76.6\%$

7.5: The Punnet square of probability

- Suppose we have the following table of young men and women with and without driver's licenses:

	Yes	No	Total
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F	486	14	500
T	977	23	1000

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- Are females less likely to be drivers?
- Probability a female is a driver: $\frac{486}{500} = 97\%$ nearly the same

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- Let's redo this using the language of events:
  - M is the event the chosen person is male
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- We need a name for this calculation, **conditional probability** $Pr(F|N) = Pr(N \cap F)/Pr(N)$ is the probability of F **given** N

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- We want to compare the probabilities of $Pr(A)$ versus $Pr(A|B)$ if they are equal then the events are **independent**

Expectations

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- Weighted averages

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- 45%, right?

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- $Pr(F) = 30\%$, and we want to find $Pr(E)$ which we calculated to be 60%, but where do we use the 50%?

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- **Bayes's Law:** $Pr(E \cap F) = Pr(F|E) \cdot Pr(E)$ – a weighted average!