Practice test

MA162-020
2010-07-01

Please see the website for a version formatted like the real exam.

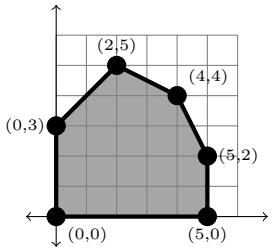
1. Describe the variables, constraints, and objective of this word problem, but you need not solve it:

A nutritionist at the medical center has been asked to prepare a special diet for some of the patients. She has decided that the meals should contain a minimum of 400mg of calcium, 10mg of iron, and 40mg of vitamin C. The hospital has two types of gruel, Blue and Green. Each ounce of Blue gruel has 30mg of of calcium, 1mg of iron, 2mg of vitamin C, and 2mg of cholesterol. Each ounce of Green gruel has 25mg of calcium, 0.5mg of iron, 5mg of vitamin C, and 5 mg of cholesterol. How many ounces of each type of gruel should be served to the patients in order to meet their special nutritional needs while minimizing their cholesterol intake?

2. Graph the feasible region for the following constraints. You are graded on three aspects: correctly drawn edges, correctly shaded region, and correctly labelled corners. List the corners and determine if the region is bounded or unbounded.

$$\left\{\begin{array}{rcl} 6x + 5y & \leq & 30 \\ 3x + y & \geq & 6 \\ x + y & \geq & 4 \\ x, y & \geq & 0 \end{array}\right\}$$

3. Find the location of the maximum value of the following objective functions on the feasible region graphed to the right:

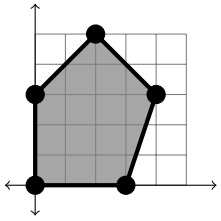


$$\left\{\begin{array}{r} P_{Feb} = 6x + 5y \\ P_{Mar} = 5x + 6y \\ P_{Apr} = 4x + 7y \end{array}\right\}$$

For instance, a Jumble Juice uses both juice concentrate and freshly squeezed juice to make its most popular drink, but alters the amount of juice concentrate and fresh juice based on various factors. Maintaining the flavor and nutritional information requires carefully staying within the feasible region, but the choice of the best mix depends on the season: in February fresh juice is quite expensive and cuts into the profit, but in April real juice is relatively cheap and more people have seconds when they taste the fresh juice, so the overall profits are higher when using real fruit juice.

4. The nutritionist in problem 1 decides to give each lucky patient meals consisting of 8 ounces of Blue gruel and 7 ounces of Green gruel. How much surplus nutrition of each type will the patients receive? How much cholesterol will they receive? Can you do better?

5. Give the inequalities defining the region to the right:



Plausible answers

1. This is problem 12 from Ch. 3.2, p. 186 of the textbook.

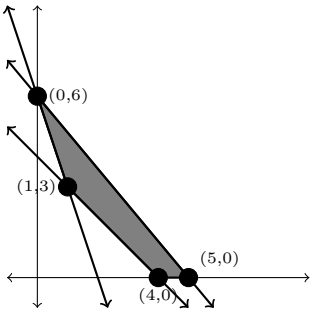
Variables:
x = ounces of Blue gruel per meal per patient
y = ounces of Green gruel " " " "

Constraints:

$$\begin{array}{rcrcrl} 30x & + & 25y & \geq & 400 & \text{(calcium)} \\ x & + & 0.5y & \geq & 10 & \text{(iron)} \\ 2x & + & 5y & \geq & 40 & \text{(vitamin C)} \\ & & x, y & \geq & 0 & \text{(sanity)} \end{array}$$

Objective:
Minimize $C = 2x + 5y$, the cholesterol intake in mg.

2. The corners are (0,6), (5,0), (4,0), (1,3). The region is bounded. Note that (0,0) is not a corner for this region, but often is for real life problems (doing nothing is often not explicitly forbidden).



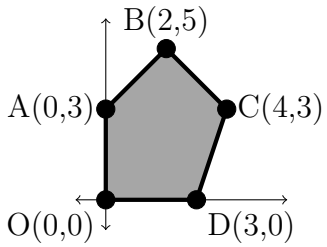
3. I suggest a table:

x	y	F	M	A
0	0	0	0	0
0	3	15	18	21
2	5	27	40	43
4	4	44	44	44
5	2	40	37	34
5	0	30	25	20

So in all three months the best choice is 4 ounces of each type.

4. If $x = 8$ and $y = 7$, then the calcium surplus is $30x + 25y - 400 = 30(8) - 25(7) - 400 = 51$ mg, the iron surplus is $x + 0.5y - 10 = 8 + 3.5 - 10 = 1.5$ mg, and the vitamin C surplus is $2x + 5y - 40 = 2(8) + 5(7) - 40 = 11$ mg. The cholesterol intake is $2x + 5y = 2(8) + 5(7) = 51$mg. One could use the method of corners to get a better answer. Lowering the Green gruel portion to $y = 6.5$ also maintains the nutritional requirements, while lowering the cholesterol.

5. This is similar to HwB1, common version numbers 1, 2, 3. You should be comfortable converting between the graphical and the algebraic views of the feasible region. Alternatively, you should be comfortable converting between corners and edges.



$$\begin{array}{rrcl} AB: & -x + y & \leq & 3 \\ BC: & x + y & \leq & 7 \\ CD: & 3x - y & \leq & 9 \\ DO: & y & \geq & 0 \\ OA: & x & \geq & 0 \end{array}$$