Worksheet Answers 2010-11-05

Name:

1. For the following polynomials, give the leading term and the end behavior.

(a) $17(x^3+4x^2-3x+2)^{12}(x-2)^4$

LT: $17(x^3)^{12}(x)^4 = 17x^{(3)(12)+4} = 17x^{40}$
$x \to +\infty$, $y \to +\infty$
$x \to -\infty$, $y \to +\infty$

(b) $17(x^3+4x^2-3x+2)^{12}(x-2)^3$

Leading Term: $17x^{(3)(12)+3} = 17x^{39}$
$x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to +\infty$

(c) $-17(x^3+4x^2-3x+2)^{12}(x-2)^4$

Leading Term: $-17x^{40}$
$x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to -\infty$

(d) $-17(x^3+4x^2-3x+2)^{12}(x-2)^3$

Leading Term: $-17x^{39}$
$x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to +\infty$

2. For the following functions give the inverse, the domains, and the ranges.

(a) $f(x) = (x+2)^2 + 3$ (not 1–1, so a **bad question**!)

Domain of f: $(-\infty, +\infty)$

Range of f^{-1}: $[-2, +\infty)$ **OR** $(-\infty, -2]$

Range of f: $[3, +\infty)$

Domain of f^{-1}: $[3, +\infty)$

Formula for $f^{-1}(x) = -2 + \sqrt{x-3}$ **OR** $f^{-1}(x) = -2 - \sqrt{x-3}$

If f is not 1–1, then it doesn't have a (single) inverse function!

(b) $f(x) = \frac{2+x}{3+x}$

Domain of f: $x \neq -3$

Range of f^{-1}: $x \neq -3$

Range of f: $x \neq 1$

Domain of f^{-1}: $x \neq 1$

Formula for $f^{-1}(x) = \frac{2-3x}{x-1}$

$$\begin{aligned} y &= \frac{2+x}{3+x} \\ y(3+x) &= (2+x) \\ 3y + xy &= 2 + x \\ xy - x &= 2 - 3y \\ x(y-1) &= (2-3y) \\ x &= \frac{2-3y}{y-1} \end{aligned}$$

$$f^{-1}(y) = \frac{2-3y}{y-1}$$

$$f^{-1}(x) = \frac{2-3x}{x-1}$$

3. Which of the following are 1–1 functions?

(a) $y = x$ ✓ (b) $y = x + 1$ ✓ (c) $y = x^2$ ✘ (d) $y = x^2 + 1$ ✘

(e) $y = |x|$ ✘ (f) $y = |x| + 1$ ✘ (g) $y = \sqrt{x}$ ✓ (h) $y = \sqrt{x} + 1$ ✓

4. Prove that the following functions are not 1–1 by giving two xs that have the same ys:

(a) $y = x^4$
$x_1 = +2$
$x_2 = -2$
$y = +16$

(b) $y = (x - 1)^4 + 1$
$x_1 = 2$
$x_2 = 0$
$y = 2$

5. Find the vertex of the following functions:

(a) $y = 2x^2 + 12x + 4$

(b) $y = 2(x + 3)^2 - 14$

$(\frac{-12}{(2)(2)}, ?) = (-3, 2(-3)^2 + 12(-3) + 4) = (-3, -14)$

Notice (a) and (b) are the same parabola. Written out in (a) and square completed in (b).

6. Which of the following functions are even or odd or both or neither? (Mark E, O, B, or N)

(a) $y = x$	O	(b) $y = x + 1$	N	(c) $y = x^2$	E	(d) $y = x^2 + 1$	E
(e) $y = x^3$	O	(f) $y = x^3 + 1$	N	(g) $y = \|x\|$	E	(h) $y = \|x\| + 1$	E
(i) $y = (x - 1)$	N	(j) $y = (x - 1)^2$	N	(k) $y = (x - 1)^3$	N	(l) $y = \|x - 1\|$	N