Example 1

A train travels from city A to city B. It leaves A at 10:00 am and arrives at B at 2:30 pm. The distance between the cities is 150 miles. What was the average velocity of the train in miles per hour (mph)?

Do you think the train was always traveling at the same speed?

Example 2

A train leaves station A at 8:00 am and arrives at station B at 10:00 am. The train stops at station B for 1 hour and then continues to station C. It arrives at station C at 3:00 pm. The average velocity from A to B was 40 mph and the average velocity from B to C was 50 mph. What was the average velocity from A to C (including stopping time)?

Example 3

Find the average rate of change of $g(x) = 2 + 4(x - 1)$ with respect to x as x changes from -2 to 5. Could you have predicted your answer using your knowledge of linear equations?

Example 4

Find the average rate of change of $k(t) = \sqrt{3t+1}$ with respect to t as t changes from 1 to 5.

Example 5

A particle is traveling along a straight line. Its position at time t seconds is given by $s(t) = 2t^2 + 3$. Find the average velocity of the particle as t changes from 0 seconds to 4 seconds.

Example 6

Let $g(x) = \frac{1}{x}$. Find a value for x such that the average rate of change of $g(x)$ from 1 to x equals $-\frac{1}{10}$.

Example 7

Find the average rate of change of $k(t) = t^3 - 5$ with respect to t as t changes from 1 to $1 + h$.

Example 8

A particle is traveling along a straight line. Its position at time t is given by $s(t) = 5t^2 + 3$. Find the instantaneous velocity of the particle when $t = 4$ seconds.

Example 9

A particle is traveling along a straight line. Its position at time t is given by $s(t) = 5t^2 + 3$. Find the velocity of the particle when $t = 2$ seconds.

Example 10

A particle is traveling along a straight line. Its position at time t is given by $s(t) = 5t^2 + 3$. Find the velocity of the particle as a function of t.

Example 11

A particle is traveling along a straight line. Its position at time t is given by $s(t) = -2t^2 + 6t + 5$.

- Find the velocity of the particle as a function of t;
- When is the velocity of the particle equal to 5 feet per second?
- When is the velocity of the particle equal to 0 feet per second?

Example 12

Find the instantaneous rate of change of $g(k) = 2k^2 + k - 1$ at $k = 3$.

Example 13

Find the instantaneous rate of change of $g(k) = 2k^2 + k - 1$ as a function of k.

Example 14

Let $f(x) = mx + b$ be an arbitrary linear function (here m and b are constants).
Prove that $f'(x) = m$.

Example 15

Let $f(x) = ax^2 + bx + c$ be an arbitrary quadratic function (here a, b and c are constants). Use the definition of the derivative to show that $f'(x) = 2ax + b$.

Example 16

Let $g(x) = 2x^2 + x - 1$. Find a value c between 1 and 4 such that the average rate of change of $g(x)$ from $x = 1$ to $x = 4$ is equal to the instantaneous rate of change of $g(x)$ at $x = c$.

Example 17

Let $g(x) = 2x^2 + x - 1$.
Find a value x_0 such that $g'(x_0) = 4$.

Example 18

Let $f(x) = x^2 + x + 14$.
What is the value of x for which the slope of the tangent line to the graph of $y = f(x)$ is equal to 5?

Example 19

Let $f(x) = x^2 + x + 14$.
What is the value of x for which the tangent line to the graph of $y = f(x)$ is parallel to the x-axis?

Example 7 (modified)

Find the average rate of change of $k(t) = t^3$ with respect to t as t changes from 1 to 2? From 1 to 1.5? From 1 to 1.1? From 1 to 1.01? From 1 to 1??