Example 1

Compute $\lim_{x \to 2} \dfrac{x^2+8}{x+2}$.

x **gets close to** 2 **from the left**				
x	1.8	1.9	1.99	1.999
$\dfrac{x^2+8}{x+2}$				

x **gets close to** 2 **from the right**				
2.001	2.01	2.1	2.2	x
				$\dfrac{x^2+8}{x+2}$

Example 2

Suppose that, instead of calculating all the values in the above tables, you simply substitute the value $x = 2$ into $\dfrac{x^2 + 8}{x + 2}$. What do you find?

Example 3

Compute $\lim_{x \to 1} [(x^2 + 4x + 3) \cdot (2x - 4)]$.

Example 4

Compute $\lim_{x \to 1} \dfrac{x^2 - 2x + 1}{x + 1}$.

Example 5

$$g(x) = \begin{cases} \dfrac{4x}{x^2+1} & \text{if } x \neq 1 \\ \\ 3 & \text{if } x = 1 \end{cases}$$

Compute $\lim_{x \to 1} g(x)$.

x	0.8	0.9	0.99	1.001	1.1	1.2
$g(x)$	1.95121	1.98895	1.9999	1.9999	1.99095	1.96721

Example 6

$$h(x) = \begin{cases} x^2 - 3 & \text{if } x > -2 \\ \\ 2x + 7 & \text{if } x \leq -2 \end{cases}$$

Analyze $\lim_{x \to -2} h(x)$.

Example 7

Analyze $\lim_{x \to 1} \dfrac{5}{(x-1)^2}$.

Example 8

Analyze $\lim_{x \to 1} \dfrac{2}{x-1}$.

Example 9

Analyze the limit $\lim_{x \to 0} \frac{2}{\sqrt{x}}$.

Example 10

Find the limit $\lim_{x \to 0} \frac{4x}{x}$.

Example 11

Find the limit $\lim_{x \to 0} \left(\frac{2}{x} + \frac{5x - 2}{x} \right)$.

Example 12

Find the limit $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$.

Example 13

Find the limit $\lim_{h \to 0} \dfrac{(h-3)^2 - 9}{h}$.

Example 14

Find the limits

$$\lim_{x\to 2^+} \frac{|3x-6|}{x-2} \qquad \lim_{x\to 2^-} \frac{|3x-6|}{x-2} \qquad \lim_{x\to 2} \frac{|3x-6|}{x-2}.$$

Example 15

Find the limits

$$\lim_{x\to+\infty} \frac{4x}{x^2+1} \qquad \lim_{x\to-\infty} \frac{4x}{x^2+1}.$$

Example 16

Find the limit $\lim_{x \to \infty} \dfrac{(2x+1)^2}{5x^2+2x+1}$.

Example 17

Find the limit of the sum as the number of terms in the sum

$$\frac{12}{10^2} + \frac{12}{10^4} + \frac{12}{10^6} + \frac{12}{10^8} + \cdots + \frac{12}{10^{2n}}$$

tends to infinity.

(**Hint:** write the sum as a decimal number first.)

Example 18

Consider the function $$f(x) = \begin{cases} x^2 - 3 & \text{if } x \leq 1 \\ 2x + B & \text{if } x > 1 \end{cases}$$

Graph the function f when $B = 4$ and $B = -1$.

Find a value of B such that the function is continuous at $x = 1$.

Example 19

Let f be the function that associates to any value of x the greatest integer less than or equal to x.
Compute the values of f at $x = 0.5,\ 1.99,\ 2,\ 2.01,\ 4.87,\ -1.5$.
Make a graph of the function f.

Compute $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.

Example 20

Let f be the function defined as follows

$$f(x) = \begin{cases} 1 & \text{if } n \leq x < n+1,\ n \text{ is an odd integer} \\ \\ 0 & \text{if } n \leq x < n+1,\ n \text{ is an even integer} \end{cases}$$

Sketch the graph of $f(x)$. Where is f discontinuous?

Example 21

Consider the function $f(x) = \sqrt{x}$. What can you say about the tangent line to the graph of f at the point $x = 0$?

Example 22

Consider the function $f(x) = |x|$. What can you say about the tangent line to the graph of f at the point $x = 0$?

Example 23

Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ \\ mx + b & \text{if } x > 2. \end{cases}$$

Find the values of m and b that make f differentiable at $x = 2$.

Example 24

Does the equation $x^4 + 8x^3 - x^2 - 4x - 1 = 0$ have a root inside the interval $(0, 1)$?

Example 7 (modified)

Find the average rate of change of $k(t) = t^3$ with respect to t as t changes from 1 to 2, from 1 to 1.5, from 1 to 1.1, from 1 to 1.01, from 1 to 1.001, and even from 1 to 1!

t	k	Δk	Δt	$\Delta k/\Delta t$
1	1	0	0	?
2	8	7	1	7
1.5	3.375	2.375	0.5	4.75
1.1	1.331	0.331	0.1	3.31
1.01	1.030301	0.030301	0.01	3.0301
1.001	1.003003001	0.003003001	0.001	3.003001
$1.000\ldots$	$\ldots$	$\ldots$	$\ldots$	$3.000\ldots$
1				3

Example 7 (modified)

We could use algebra to find the average rate of change from $t = 1$ to $t = 1 + h$ of $k(t) = t^3$:

$$\frac{\Delta k}{\Delta t} = \frac{k(1+h) - k(1)}{(1+h) - 1}$$

$$= \frac{(1+h)^3 - (1)^3}{h}$$

$$= \frac{(1 + 3h + 3h^2 + h^3) - (1)}{h}$$

$$= \frac{3h + 3h^2 + h^3}{h}$$

$$= 3 + 3h + h^2 \qquad \to 3 + 3(0) + (0)^2 = 3$$

Example 15

$$\lim_{x\to+\infty} \frac{4x}{x^2+1} = 0^+ \qquad \lim_{x\to-\infty} \frac{4x}{x^2+1} = 0^-$$

x	$\frac{4x}{x^2+1}$	x	$\frac{4x}{x^2+1}$
10	$\frac{40}{101} \approx 0.4$	-10	$\frac{-40}{101} \approx -0.4$
100	$\frac{400}{10001} \approx 0.04$	-100	$\frac{-400}{10001} \approx -0.04$
1000	0.004	-1000	-0.004
10000	0.0004	-10000	-0.0004
100000	0.00004	-100000	-0.00004
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$10000\ldots$	$0.000\ldots$	$-10000\ldots$	$-0.000\ldots$
$+\infty$	$+0$	$-\infty$	-0

Quiz Answer

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = 8$$

Table method:

x	$\frac{x^2-16}{x-4}$
5	9
4.5	8.5
4.1	8.1
4.01	8.01
4.001	8.001
$\vdots$	$\vdots$
$4.000\ldots$	$8.000\ldots$
4	8

Algebra method:

$$\frac{x^2 - 16}{x - 4} = \frac{(x-4)(x+4)}{(x-4)} = x + 4$$

so

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} (x+4) = (4+4) = 8$$