

## Exam 2: Fair-division

### Part I: Perception of value

(Practice) 2011-02-14

1. Three players Alvin, Byron, and Chiador are sharing three slices of cake. The following table gives the value of each slice in the eyes of the players.

	1st slice	2nd slice	3rd slice
Alvin	\$6.00	\$7.00	\$8.00
Byron	\$6.00	\$6.50	\$5.50
Chiador	\$8.00	\$6.25	\$6.75

(a) Which slices would Alvin view as fair shares (so that he would not feel cheated if he received such a slice)?

Alvin thinks the loot is worth  $6+7+8 = 21$  dollars.  
 He feels his fair share is at least  $21/3 = 7$  dollars.  
 Both the second and third slice would satisfy him.

2nd + 3rd

(b) Which slices would Byron view as fair shares?

Byron wants at least  $(6+6.5+5.5)/3 = 6$  dollars. Both the first and second are good enough.

1st + 2nd

(c) Which slices would Chiador view as fair shares?

Chiador wants at least  $(8+6.25+6.75)/3 = 7$  dollars. Only the first slice will do.

1st

## Part II: You cut - I choose

2. Desmond likes strawberry jelly about three times as much as grape jelly. He picks up sandwiches for himself and his new lab partner, but notices he has one grape and one strawberry sandwich.

(a) If he lets his lab partner pick a sandwich, why might Desmond feel cheated at the end?

His partner might pick the strawberry sandwich, leaving Desmond with grape. Desmond's share would only be  $\frac{1}{4}$  of the loot in his eyes. He wants at least  $\frac{1}{2}$ .

(b) If he cuts both the strawberry and the grape sandwiches exactly in half, then he could make each division have half of each sandwich. Both divisions look basically identical. Could either player reasonably feel cheated? Could either player feel they did better than expected?

If both portions are truly identical, then neither player could feel either cheated or like they did better than expected: both portions are exactly 50%.

However we know nothing of the partner's value system. The portions may not be identical in his eyes.

(c) Desmond decides to make one serving that has only  $\frac{2}{3}$  of the strawberry, and the other serving that is other  $\frac{1}{3}$  of the strawberry and the entire grape. Could Desmond feel cheated? Could his lab partner feel like Desmond is being generous?

If the two sandwiches total to \$4, Desmond thinks the strawberry is worth \$3 and the grape \$1. The first portion is worth  $(\frac{2}{3})$  of \$3, that is \$2, and the second

portion is worth  $(\frac{1}{3})$  of \$3 plus \$1, that is \$2.

Both portions have equal value to Desmond.

If the lab partner likes grape better, then the <sup>2nd</sup> portion will appear very generous.

### Part III: Last diminisher

3. Adam Sandler, Bruce Willis, Chuck Norris, and Dwayne Johnson are going to play the card game War. Before playing, they must divide the deck between them. They don't want to take the time to count out 52 cards, so they use the method of Last diminisher to quickly divide the cards among them. Each player wants to have as many cards as possible.

(a) Adam selects a claim. To Bruce, it appears to be 33% of the deck. What should he do?

Bruce cannot guarantee he can keep the claim, so he wants it to be exactly a fair share (either for him to keep, or for Chuck or Duane to claim).

He should reduce it to 25%.

(b) On Chuck's turn, the claim appears to be 20% of the deck. What should Chuck do?

Chuck should pass. 20% is too small to be his fair share.

(c) When it is Dwayne's turn, it appears to Dwayne that the claim is 28.74812% of the deck. What should he do?

While such precision is impressive it is suspicious. However  $28\% > 25\%$ , so the claim is at least fair. Since Dwayne does decide if the claim is his, he should just take it as is.

(d) In round 2, how much of the remaining deck should Adam claim?

With 3 players the perfect play is to claim  $\frac{1}{3} \approx 33\%$ , that is, he should play honestly.

If the previous round went ~~well~~ well, the claim should be 13 cards. However, no matter how the first round went, Adam should claim  $\frac{1}{3}$  of what's left.

## Part IV: Sealed Bids

4. Sally and Marcy have been roommates and BFF. However, the semester is ending and they need to split up their furniture they bought together. They feel each is owed 50% of the loot, but they have different ideas of what each item is worth. They still have some cash, and so decide the method of sealed bids is appropriate. They open up the bids to reveal the following:

	Bieber poster	Velvet Elvis	Lava Lamp	Giant Pillow	String Lights	White-board
Sally	<u>\$10</u>	<u>\$6</u>	\$5	\$3	<u>\$10</u>	\$10
Marcy	\$2	\$5	<u>\$6</u>	<u>\$10</u>	\$5	<u>\$12</u>

(a) Which items does Sally get?

*Sally gets the poster, the elvis, and the lights. She had the highest bid on each of these.*

(b) Who pays whom, and how much do they pay?

*Sally pays in  $10 + 6 + 10 = 26$ . Sally takes out  $(10 + 6 + 5 + 3 + 10 + 10) / 2 = 22$ . Marcy puts in  $6 + 10 + 12 = 28$ . Marcy takes out  $(2 + 5 + 6 + 10 + 5 + 12) / 2 = 20$ . There is 12 leftover, so each gets 6. Sally gets \$2 from Marcy.*

(c) How much does Marcy think her fair share is worth?

*→ Marcy wants half of  $2 + 5 + 6 + 10 + 5 + 12$ , that is \$20.*

(d) How much does Marcy end up getting in her opinion?

*Marcy gets Lamp (6) + Pillow (10) + Board (12), but pays \$2, so  $6 + 10 + 12 - 2 = \$26$  total. \$6 more than expected.*