### **Practice Exam**

### Part I: Vocabulary

- 1. Match the word with its definition:
- (a) Euler path

(1) The number of times the vertex appears in the list of edges. The number of edges adjacent to the vertex, where loops count twice.

(b) Euler circuit

(2) An ordering to the edges, so each edge is adjacent to the next one, and the first and last edges are adjacent. A tracing that starts where it finishes.

(c) Connected

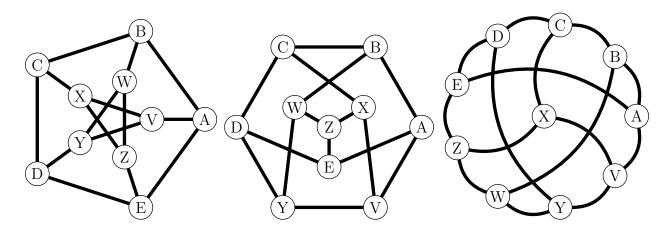
(3) An edge such that removing it results in a disconnected graph.

(d) Degree

(4) An ordering to the edges, so each edge is adjacent to the next one, and the first and last edges are not adjacent. A tracing that starts and finishes in different places.

(e) Bridge

- (5) A graph such that between any two vertices, there is a sequence of edges, each adjacent to the next, that starts at one of the vertices, and ends at the other.
- 2. Here are three graphs. Circle one and answer the following questions:



- (a) List the vertices:
- (b) List the edges (alphabetically, please):
- (c) What are the degrees of the vertices?

#### Part II: Euler's little theorem

1. Construct a graph with vertices of degree 3, 3, 3, 3 or explain why no such graph exists.

2. Construct a graph with vertices of degree 2, 2, 2 or explain why no such graph exists.

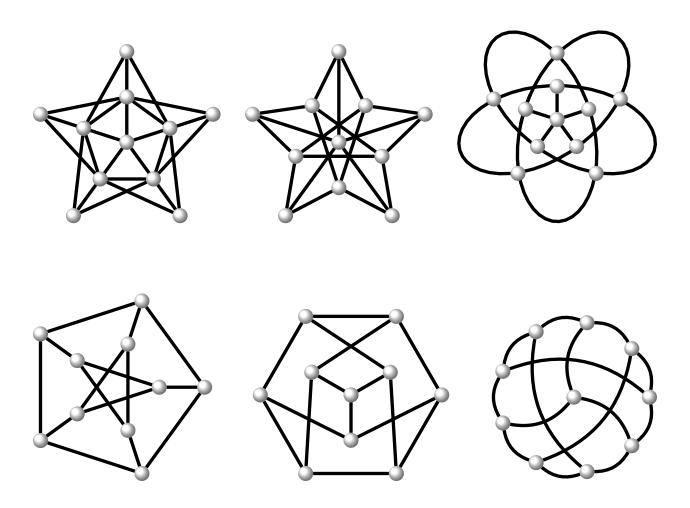
3. Construct a graph with vertices of degree 3, 3, 3 or explain why no such graph exists.

4. Construct a graph with vertices of degree 4, 4, 4, 4, 4, 4 or explain why no such graph exists.

5. Construct a graph with vertices of degree 1, 2, 3, 2 or explain why no such graph exists.

# Part III: Euler's kingly mountain theorem

1. Decide whether each graph has an Euler circuit, an Euler path, both, or neither.



2. How did you decide for each graph?

# Part IV: Fleury's algorithm

1. For each graph label the edges  $1,2,3,\ldots$  in order of an Euler circuit or Euler path.

