PRACTICE EXAM

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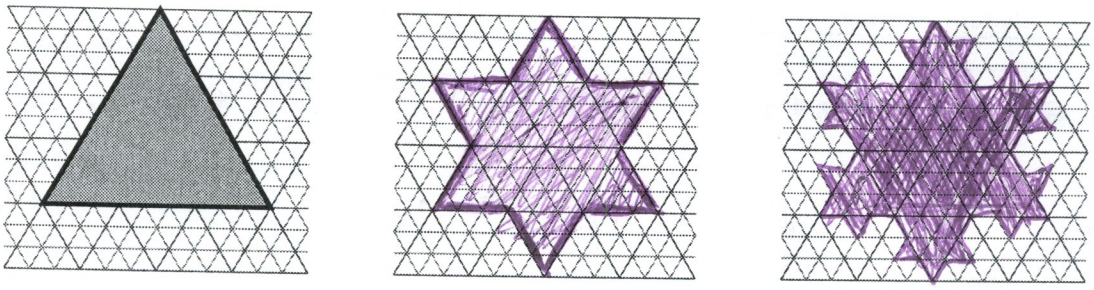
MA111-009

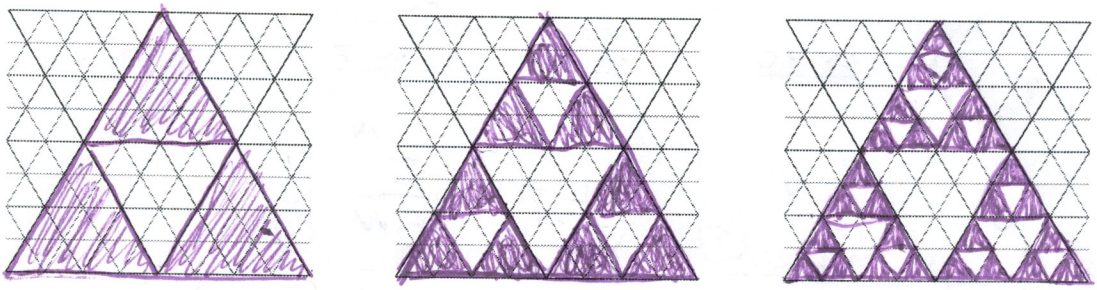
Part I: Draw a Koch snowflake

The seed is a triangle △. Replace each _____ with _/_. Draw the next 2 stages and shade the inside.



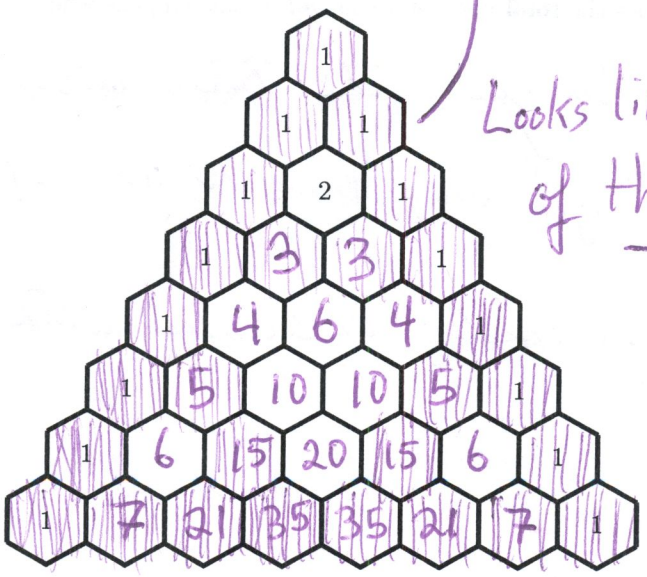
Part II: Draw a Sierpinski triangle

The seed is a triangle ▲. Divide each triangle into 4 equal triangles and discard the middle one. Draw the next 3 stages, shading the remaining part.



Part III: Adding up numbers is lots of fun

Fill each hexagon with the sum of the two numbers above it. Lightly shade the odd numbers.

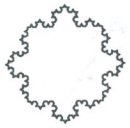
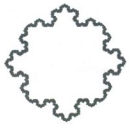


Looks like Step 2 of the Sierpinski Triangle

Which fractal does this resemble?

Part IV: Find the perimeter of a Koch squareflake

Begin with a square whose sides have length 1. Replace each _____ with _/_.

Step	Picture	Number of edges	Edge length	Total perimeter
Seed		4	1	4
Step 1		16	$\frac{1}{3}$	$\frac{16}{3}$
Step 2		64	$\frac{1}{9}$	$\frac{64}{9}$
Step 3		256	$\frac{1}{27}$	$\frac{256}{27}$
⋮				
Step N		$(4^N)(4)$	$\left(\frac{1}{3^N}\right)$	$\left(\frac{4^N}{3^N}\right)(4)$

↑ 4 times as many edges, each $\frac{1}{3}$ as long!

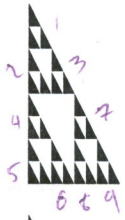
As N gets larger, how does the total perimeter change? What happens when N goes to infinity?

As N gets larger, the total perimeter gets larger.
(33% larger every time N goes up by 1)

The total perimeter increases without bound. $P \to \infty$

Part V: Find the area of a Sierpinski right triangle

The Sierpinski Triangle is formed recursively, starting with a filled-in triangle. For every triangle remaining, divide the triangle into 4 equal sub-triangles and remove the middle. Now repeat the previous step on the remaining triangles. The starting triangle is a right triangle with base of length 1, and height of length 2.

Step	Picture	# New holes	Area of each	Area left
Start		0	-	1
Step 1		1	$\frac{1}{4}$	$\frac{3}{4}$
Step 2		3	$\frac{1}{16}$	$\frac{3}{4} - \frac{3}{16} = \frac{9}{16}$ (also 9 black △ out of 16 total)
Step 3		9	$\frac{1}{64}$	$\frac{27}{64}$
Step 4		27	$\frac{1}{256}$	$\frac{81}{256}$
Step N		3^{N-1}	$\frac{1}{4^N}$	$\frac{3^N}{4^N}$

As N gets larger, what happens to the area? If N goes to infinity, what is the "final" area?

As N increases, the area decreases. (25% for every N)
As N increases, the area decreases to 0. $A \to 0$

Part V: Free response

1. What is the most useful thing you learned this semester?

Either Rachel's survivor method of voting and how it encourages mediocre candidates, or Twyla's method of sealed bids which divides the bonus very simply.

2. What is the prettiest thing you learned this semester?

Either the five sided snowflakes or Jason's exercise on symmetry in music. I was especially impressed by the clapping exercise which layered competing rhythms that immediately changed to cooperation.

3. What is the hardest thing you learned this semester?

To give structured, achievable assignments. I expect students to explore, but by dropping them in a jungle with a broken compass. By giving more structured activities more of the class was brave enough to explore.

4. Why do people study mathematics?

Some study it out of necessity: hard problems need the power of mathematics to find solutions. Some study it out of reverence: the hidden structures of mathematical objects point to a guiding principle in all of reality.
I study it because it is fun.