DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam -1 December 16, 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure that your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations, but you must show your work to receive credit.
Put your answers in the answer boxes provided, and show your work.
If your answer is not in the box or if you have no work to support your answer, you will receive no credit.
The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	15	
2	15	
3	8	
4	10	
5	16	
6	12	
7	12	
8	12	
Total	100	

Please fill in the information below. NAME: JACK

Section: 999

Last four digits of Student ID: 9999

1. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

Tanya sells two nutritional supplements called "Energy" and "Vitality".

X Each bottle of "Energy" sells for $ 20, requires 4.5 ounces of herbal mixture and 3.5 ounces of nutritional syrup.

y Each bottle of "Vitality" sells for $ 18, requires 3.5 ounces of herbal mixture and 4.5 ounces of nutritional syrup.

RHS Tanya has 225 ounces of herbal mixture and 275 ounces of nutritional syrup on hand.

Set up a LPP whose solution will determine how many bottles of "Energy" and how many bottles of "Vitality" should she make to maximize her profit.

i) Define and explain all the variables you use.

Variables are:

x = how many bottles of "Energy" to make

y = " " " " "Vitality" to make

ii) Now describe the LPP explicitly by writing **all the inequalities** and specifying the objective function.

Maximize: Profit $P =$ $\$20x + \$18y$

Subject to:

$4.5x + 3.5y \leq 225$ (herbs)

$3.5x + 4.5y \leq 275$ (syrup)

$x \geq 0, y \geq 0$

iii) The initial Simplex tableau will be:

x	y	H	S	P	RHS
4.5	3.5	1	0	0	225
3.5	4.5	0	1	0	275
-20	-18	0	0	1	0

2. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.

$2x + 3y \leq 18$

$y \leq x + 2$

$x \geq 0$

$y \geq 0$

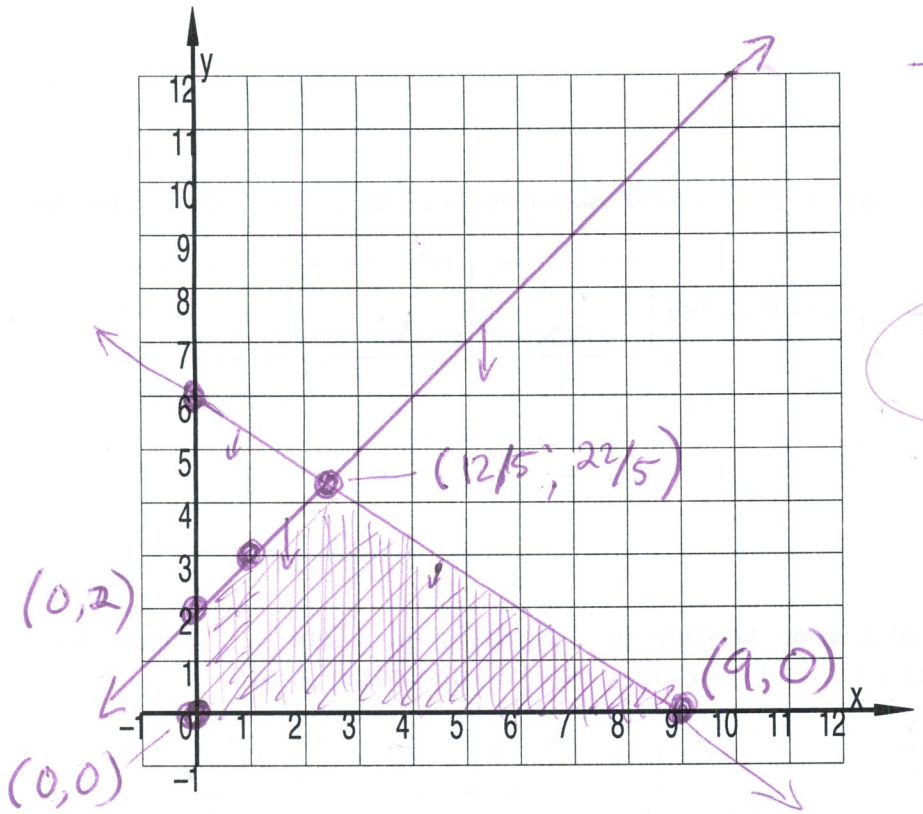
x	y
0	6
9	0
12/5	22/5
0	2
1	3
12/5	22/5

$2x + 3(x+2) = 18$

$5x = 12$

$x = 12/5$

$y = \frac{12}{5} + 2 = \frac{22}{5}$



Corners

x	y	P
0	0	0
0	2	4
12/5	22/5	44/5 ≈ 8.8
9	0	18

ii) Find the maximum value of the function, $P = 2x + 2y$ on the region.

Answer: $P =$ 18 at $x =$ 9 , $y =$ 0 .

3. Here is a final tableau associated with a maximal LPP.

x	y	z	s	t	u	P	constants
0	0	1	3	−2	−5	0	11
0	1	0	1	1	5	0	6
1	0	0	1	1	4	0	2
0	0	0	3	6	29	1	17

(a) Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.

Value of P= 17 (x , y , z)=(2 , 6 , 11)

Read by rows using pivots

(b) Recall that the solution of the **dual minimization problem** can be read from the same tableau. Determine the solution of the dual minimization LPP.

Value of C= 17 (s , t , u)=(3 , 6 , 29)

Read columns by Label

4. Set this problem up, by stating the chosen variables, the equations to be solved and **the initial augmented matrix. Do not solve the problem.**

(A) Oliver has a farm with 240 acres. He is planning on planting tomatoes, corn and wheat and wants to decide how many acres to use for each crop.

(B) Oliver's County agent Hank has advised him to plant as many acres of wheat as acres of tomatoes and corn together.

(C) Oliver has a total of 11,580 dollars available for planting expenses and knows that tomatoes cost 28 dollars per acre, corn costs 63 dollars per acre amd wheat costs 51 dollars per acre.

Oliver is wondering if it is possible to plant all the available acres while using up all the funds and following Hank's advice.

His wife Lisa tells him that all he has to do is to write and solve some linear equations.

Help Oliver by setting up equations for him and writing the initial augmented matrix.

Use x,y,z to denote the number of acres of "tomates", "corn" and "wheat" respectively.

The equations to be solved are:

$x + y + z = 240$ (A)

$z = x + y$ (B) $\longrightarrow -x - y + z = 0$

$28x + 63y + 51z = 11580$ (C)

The augmented matrix is:

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & 1 & 1 & 240 \\ -1 & -1 & 1 & 0 \\ 28 & 63 & 51 & 11580 \end{array}\right)$$

5. (i) Consider the following system of linear equations.

$$x + 3y + z = 2$$

$$3x + 8y + 7z = 10$$

Write down the augmented matrix for this system of equations.

$$\begin{matrix} x & y & z & RHS \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 3 & 8 & 7 & 10 \end{array}\right)$$

Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used. You may need more or fewer boxes than provided. Use the empty space on the back of the previous page if you need additional space.

$$R_2 - 3R_1 \longrightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 4 & 4 \end{array}\right)$$

REF

$$\begin{matrix} R_1 + 3R_2 \\ -R_2 \end{matrix} \longrightarrow \begin{matrix} x & y & z & RHS \end{matrix}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 13 & 14 \\ 0 & 1 & -4 & -4 \end{array}\right)$$

RREF

(ii) Using above calculations, determine all the solutions to the system of equations in x, y, z given above.

$x = 14 - 13z$, $z = FREE$

$y = -4 + 4z$,

6. A group of 171 citizens were surveyed about their views on the proposals for "deficit reduction" and "continued tax-cuts for the rich".

It was found that 81 citizens were for the "deficit reduction" and 26 were for "continued tax-cuts for the rich". In the survey 70 citizens announced that they did not approve of either of these proposals and prefer to work on improving the economy by other means.

D: 81
C: 26
neither: 70

total: 171
either: 171 − 70 = 101

(i) Estimate the probability that a random citizen likes at least one of the two proposals.

101/171

$\frac{171-70}{171}$

59%

(ii) Estimate the probability that a random citizen approves both the proposals.

6/171

$\frac{D \quad C \quad D\cap C}{81+26-101}$
$\frac{}{171}$

3.5%

(iii) Estimate the probability that a random citizen approves the proposal for "deficit reduction" but not the "continued tax-cuts for the rich".

75/171

$\frac{D \quad D\cap C}{81-6}$
$\frac{}{171}$

44%

7. Mike is developing a new toy called "Oracle" which gives advice. You formulate a question in your mind and press a button on the top . It promptlly displays one of the eight numbers $1, 2, 3, 4, 5, 6, 7, 8$. You then read the advice of the Oracle in a table by looking up the number.

Mike is trying to test whether his toy is giving all the 8 numbers with equal probability. So he tests his toy many times and keeps record of outcomes.

Let A be the event that the toy shows a number bigger than or equal to 4, i.e. one of $4, 5, 6, 7, 8$.

Let B be the event that the toy shows a number less than or equal to 5, i.e. one of $1, 2, 3, 4, 5$.

Mike's experiments estimate that $P(A) = 59\%$ and $P(B) = 63\%$

Answer the following questions **based on these experimental observations.**

(i) What is the probability that displayed number is 4 or 5 ? $\boxed{22}$ %.

$A \cap B = \{4, 5\}$

$A \quad B \quad A \cup B \quad A \cap B$

$59\% + 63\% - 100\% = 22\%$

(ii) What is the probability that displayed number is 1 or 2 or 3 ? ~~37~~ %. 41%

~~$A - B = \{1, 2, 3\}$~~

$B - A = \{1, 2, 3\}$ ~~$A \quad B$~~ $A \cup B$ ~~$A - B$~~ $B - A$

63% ~~59%~~ $- 22\% = 37\%$ 41%

(iii) Mike concludes that his toy is not showing all numbers with equal probability, since in that case, the probability of the numbers 4, 5 showing would be equal to $\boxed{25}$ %.

$\frac{2 \text{ numbers}}{8 \text{ numbers}}$

8. A company finds that it does not have enough funds to give its usual healthy annual bonuses. So, to spread at least some cheer, it is offering a few "dream prizes", which are awarded by random draws.

As a result, 88 of its 230 managers won "dream prizes" and 130 of its 590 non managers also won the "dream prizes. Naturally, employess who did not get the prizes were not convinced of the fairness of the prize scheme.

Calculate the following probabilities to decide on this fairness issue.

(i) What is the probability that a random employee of the company is a manager?

$\boxed{230/820}$ 28% $230/(230+590)$

(ii) What is the probability that a random employee of the company won the prize?

$\boxed{218/820}$ 26.6% $(88+130)/(230+590)$

(iii) Are the events "being a manager" and "winning a prize" independent? $\boxed{\text{No}}$

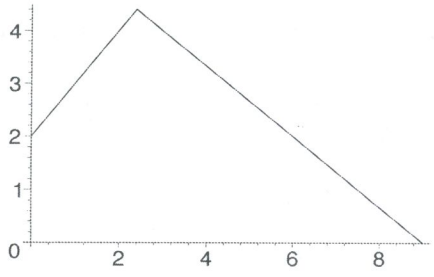
Important. Note that the shareholders have declared that they consider two events A, B to be independent only when $P(A)P(B)$ differs from $P(A \cap B)$ by less than 0.01.

$\left(\frac{230}{820}\right)\left(\frac{218}{820}\right) \neq \frac{88}{820}$

7.46% $\neq$ 10.73%

1 Answer Key for exam4v-1

1. ◇ x= energy bottle count
   y= vitality bottle count
   ◇ $P = 20\,x + 18\,y \quad 0 \le x \quad 0 \le y \quad 9/2\,x + 7/2\,y \le 225 \quad 7/2\,x + 9/2\,y \le 275$
   ◇ **Grading:**15 pt.: Variables 2 pt. Obj. fn. 3 pts.
   Ineq.(6 pt.) 2 pts. each including positivity Tableaux 4 pt.

2. 

   ◇ $P = \frac{68}{5}$ at $x = \frac{12}{5}$ $y = \frac{22}{5}$. $[[x = 9, y = 0], [x = \frac{12}{5}, y = \frac{22}{5}], [x = 0, y = 2], [x = 0, y = 0]]$
   ◇ **Grading:**15 pt. Lines (6 pt.) 3pt. each
   Corners (6pt.) 2 pt. each Fn. evaluation (3 pt.)

3. ◇ (a) P= 17 (x,y,z) $[2, 6, 11]$
   ◇ (b) C= 17 (s,t,u) $[3, 6, 29]$
   ◇ Grading: 8pt. One point for each answer
   fn. values and var. values.

4. x = tomato acres y = corn acres z = wheat acres
   ◇
$$x + y + z = 240$$
$$28\,x + 63\,y + 51\,z = 11580$$
$$z = x + y$$
   ◇ Grading: 10 pt. Equations (8 pt.)
   3 per eqn. except 2 for cost eqn. Augmented matrix (2 pt.)

5. ◇
$$\begin{array}{cccccccc} 1 & 3 & 1 & 2 & 1 & 0 & 13 & 14 \\ 3 & 8 & 7 & 10 & 0 & 1 & -4 & -4 \end{array} \quad [x, y, z] = [-13\,z + 14, 4\,z - 4, z]$$
   Grading 16 pt.; Matr
   ◇ Backsub or extra row operation: 4 pt. Final answer- must
   Irregular steps with wrong answers lose backsub points
   One point off for each mistake or wrong or missing notation until gone.

6. ◇ (i)Vote for at least one= $[\frac{101}{171}$, "59.06", "*percent*"] (ii)Vote for both = $[\frac{2}{57}$, "3.51", "*percent*"] (iii
   ◇ Only deficit reduction = $[\frac{25}{57}$, "43.86", "*percent*"]
   Grading: 12 pt. Each part 4 pt.
   ◇ With correct formula, only 1 off for wrong answer.
   For miscopied but not misused numbers 2 pts.