MA162: Finite mathematics

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January 31, 2011

Schedule:

- HW A2 is due Today, Jan 31st, 2011.
- HW A3 is due **Sunday**, Feb 6th, 2011.
- Exam 1 is Monday, Feb 7th, 5:00pm-7:00pm.
- HW B1 is due Monday, Feb 21st, 2011.

Today we will cover 2.3 and pages 7-8 of the appendix: degeneracy and RREF

Appendix: Very efficiently solving systems

- We managed to solve some fairly big systems last time using our **new** number crunching skills.
- Mostly it was repetitive, routine, soothing.
- But near the end we stopped the number pushing and revived the variables, which totally harshed my zen.
- Today we learn to finish the easy way

Appendix: Cleaning above as well as below

- A matrix is in **REF** if no column (left of the bar) has two pivots
- This means that below and to the left of each pivot are zeros
- A matrix is in **RREF** if
  - it is in REF,
  - there are only zeros above pivots, and
  - pivots are equal to 1

Appendix: How to clean

- If a matrix is in REF, then a **possible target** is a non-zero number above a pivot

- We choose the right-most column with a possible target, and then choose the bottom-most possible target in that column

- The row operation is the same as before:

$$R_{target} - \frac{target}{active} \cdot R_{active}$$

- If a pivot is not equal to one, then we can divide the whole row by the pivot

Appendix: Example

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 15 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2} R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

RREF

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2.3: What if things go wrong?

- Is this matrix in REF? RREF?

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- What could we do to fix it?
  - Row 2 can only make row 1 worse and vice versa!
  - Row 3 cannot do anything at all!
- Let's write it out in variables, and see what is going on:

$$x + y = 1 \qquad z = 1 \qquad 0 = 0$$

- Well that is not too bad? $x = 1 - y$, y is free, $z = 1$.
- We can read this right from the matrix
- We do say this matrix is in REF and RREF

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2.3: Free variables

- If a column (for a variable) has no pivot, then that variable is **free**
- Be careful when reading the answer off the matrix
  110|1 means $x + y = 1$, so $x = 1 - y$
- If a variable is free, then (assuming there are any solutions) there are **infinitely many solutions**
- What does "no solution" look like in matrix format?

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- **No solution**, **inconsistent**
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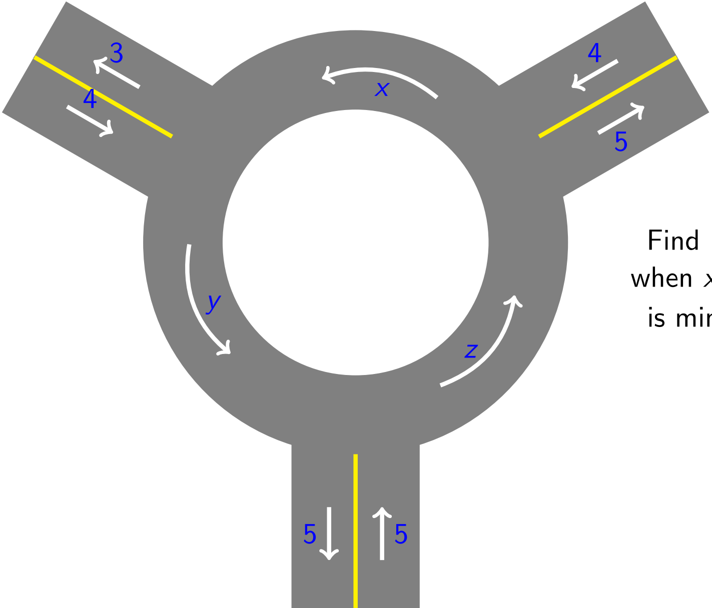
Roundabout: Smaller #5



Find (x, y, z)
when $x + y + z$
is minimized.

Line-about: same thing with lines