MA162: Finite mathematics

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SCHEDULE:

- Expect exams available by Friday, Feb 11th, 2011.
- HW B1 is due Monday, Feb 21st, 2011.
- HW B2 is due Monday, Feb 28th, 2011.
- HW B3 is due Sunday, Mar 6th, 2011.
- Exam 2 is Monday, Mar 7th, 5:00pm-7:00pm.
- My office hours are Monday, Wednesday 3pm and Friday 9am; all in mathskeller

Today we will cover 2.4 and some of 2.5: matrix arithmetic

2.4: Matrix arithmetic

- We saved time and worked more efficiently by converting systems of equations to matrices
- We treated each row of a matrix like a single (fancy) number,
- We added rows, subtracted rows, and multiplied rows by numbers
- Now we learn to treat entire matrices as (very fancy) numbers
- Today we will **add**, **subtract**, **multiply by numbers**, and **multiply**
- Next week we will divide; in chapter 3 we will solve real problems

2.4: Matrix size

- A matrix is a rectangular array of numbers, like a table
- A matrix has a **size**: the number of **rows** and **columns**
- A 2×3 matrix has 2 rows, and 3 columns like:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- A 1×4 matrix has 1 row and 4 columns like:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

- A 1×1 matrix has 1 row and 1 column like:

$$\begin{bmatrix} 19 \end{bmatrix}$$

2.4: Matrix equality

- Two matrices are equal if they have the same size, and the same numbers in the same place
- If these two matrices are equal,

$$\begin{bmatrix} 1 & x \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} y & 2 \\ 3 & 4 \end{bmatrix}$$

  then $x = 2$ and $y = 1$

- None of these matrices are equal to each other:

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

2.4: Matrix addition

- We can add matrices if they are the same size by adding entry-wise:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 22 \\ 23 & 24 \end{bmatrix} = \begin{bmatrix} 11+21 & 12+22 \\ 13+23 & 14+24 \end{bmatrix} = \begin{bmatrix} 32 & 34 \\ 36 & 38 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} + \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \begin{bmatrix} 22 & 24 & 26 & 28 \\ 30 & 32 & 34 & 36 \\ 38 & 40 & 42 & 44 \end{bmatrix}$$

- Different shaped matrices are not added together:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \textbf{nonsense; undefined}$$

2.4: Matrix subtraction

- We can subtract matrices if they are the same size:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} - \begin{bmatrix} 21 & 22 \\ 23 & 24 \end{bmatrix} = \begin{bmatrix} 11-21 & 12-22 \\ 13-23 & 14-24 \end{bmatrix} = \begin{bmatrix} -10 & -10 \\ -10 & -10 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} - \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \begin{bmatrix} -20 & -20 & -20 & -20 \\ -20 & -20 & -20 & -20 \\ -20 & -20 & -20 & -20 \end{bmatrix}$$

- Different shaped matrices are not subtracted from one another:

$$\begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} - \begin{bmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \end{bmatrix} = \textbf{nonsense; undefined}$$

2.4: Scalar multiplication

- We can multiply a matrix by a number (a **scalar**):

$$5 \cdot \begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} = \begin{bmatrix} 5 \cdot 11 & 5 \cdot 12 \\ 5 \cdot 13 & 5 \cdot 14 \end{bmatrix} = \begin{bmatrix} 55 & 60 \\ 65 & 70 \end{bmatrix}$$

- Big matrices are no harder, just more of the same:

$$3 \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 & 12 \\ 15 & 18 & 21 & 24 \\ 27 & 30 & 33 & 36 \end{bmatrix}$$

- There is no restriction on size of the matrix,
  but remember we aren't multiplying two matrices yet:

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = ???$$

2.5: Matrix-matrix multiplication

- Matrix-matrix multiplication can be defined several ways
- Only one way is particularly useful to us in this class
- A simple example: We want to write down

$$\begin{aligned} 1x + 2y &= 3 \\ 4x + 5y &= 6 \end{aligned}$$

- Using our multiplication this becomes:

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

- Cleanly separates the variables and the numbers,
  keeps the $+$ and $=$ signs, so lets us be more flexible

2.5: Matrix-matrix multiplication

- To find the **top-left** entry of the product,
  we multiply the **top** row by the **left** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & ? \\ ? & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 58 & ? \\ ? & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **top-right** entry of the product, we multiply the **top** row by the **right** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & 1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 \\ ? & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ ? & ? \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ ? & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **bottom-left** entry of the product, we multiply the **bottom** row by the **left** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1\cdot 7 + 2\cdot 9 + 3\cdot 11 & 1\cdot 8 + 2\cdot 10 + 3\cdot 12 \\ 4\cdot 7 + 5\cdot 9 + 6\cdot 11 & ? \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ 28 + 45 + 66 & ? \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & ? \end{bmatrix}$$

2.5: Matrix-matrix multiplication

- To find the **bottom-right** entry of the product, we multiply the **bottom** row by the **right** column

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1\cdot 7 + 2\cdot 9 + 3\cdot 11 & 1\cdot 8 + 2\cdot 10 + 3\cdot 12 \\ 4\cdot 7 + 5\cdot 9 + 6\cdot 11 & 4\cdot 8 + 5\cdot 10 + 6\cdot 12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 + 18 + 33 & 8 + 20 + 36 \\ 28 + 45 + 66 & 32 + 50 + 72 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

2.5: Bracelet Franchises

- Our bracelet manufacturers are exploring new markets
- Different bracelets are more popular in different areas, so we researched the demand for each:

	Frankfort	Georgetown	Hazard
Aurora	100	200	300
Babylon	400	200	0
Camelot	100	200	600

- Each location manufacturers locally, How much material to send to each?

2.5: More tabulated data

- Each bracelet requires different amounts of each part:

	Aurora	Babylon	Camelot
Beads	10	4	6
Wires	1	2	3
Clasps	1	1	1

	Frankfort	Georgetown	Hazard
Aurora	100	200	300
Babylon	400	200	0
Camelot	100	200	600

2.5: A single question

- How many beads does our Frankfort branch need?

	Aurora	Babylon	Camelot
Beads	10	4	6
Wires	1	2	3
Clasps	1	1	1

	Frankfort	Georgetown	Hazard
Aurora	100	200	300
Babylon	400	200	0
Camelot	100	200	600

$$10 \cdot 100 + 4 \cdot 400 + 6 \cdot 100 = 3200$$

2.5: Full summary table

- How to find all of the data?
- Multiplying these two tables as matrices gives a full table of how much is needed by each franchise:

	Frankfort	Georgetown	Hazard
Beads	3200	4000	6600
Wires	1200	1200	$1 \cdot 300 + 2 \cdot 0 + 3 \cdot 600 = 2100$
Clasps	600	600	900

	Aurora	**Babylon**	**Camelot**
Beads	**10**	**4**	**6**
Wires	**1**	**2**	**3**
Clasps	**1**	**1**	**1**

	Frankfort	**Georgetown**	**Hazard**
Aurora	**100**	**200**	**300**
Babylon	**400**	**200**	**0**
Camelot	**100**	**200**	**600**

Homework: Tricky homework type

- Struggling is good; don't worry, don't give up
- Don't worry about the inverses yet, we will cover them next week
- Some of the problems are easy; you can do them today
- Some are tricky and require you to use the basic skills we learned today in new ways:

  If $\begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} x & 3 \end{bmatrix} = \begin{bmatrix} 5 & y \end{bmatrix}$, then what are x and y?

  $1 + x = 5$ so $x = 4$, $2 + 3 = y$ so $y = 5$

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