# MA162: Finite mathematics

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University of Kentucky

February 16, 2011

Schedule:

- HW B1 is due Monday, Feb 21st, 2011.
- HW B2 is due Monday, Feb 28th, 2011.
- HW B3 is due Sunday, Mar 6th, 2011.
- Exam 2 is Monday, Mar 7th, 5:00pm-7:00pm.

Today we will cover 3.1: graphing inequalities

# Exam 2: Overview

- 35% Ch. 2, Matrix arithmetic (HWB1):
  - Sizes
  - Addition, subtraction, multiplication by a number
  - Matrix-matrix multiplication and matrix to a power
  - Matrix inverse ($2 \times 2$ and $3 \times 3$)
  - Solving system given inverse
- 30% Ch 3, **Graphical inequalities** and LPP (HWB2)
- 35% Ch 4, Simplex algorithm (HWB3)

# 3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options?

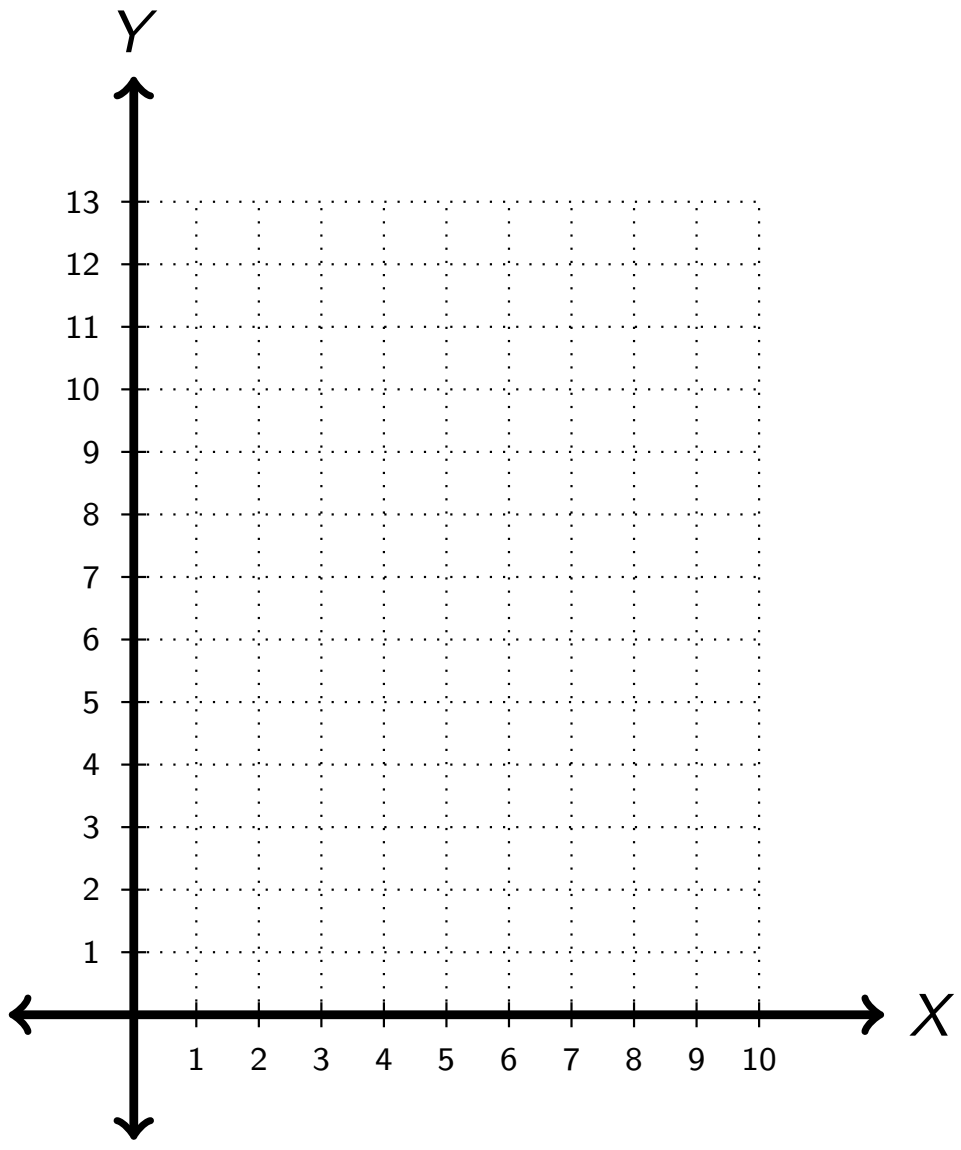
$$200x + 100y = 1000$$

# 3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options? Don't have to spend it all!

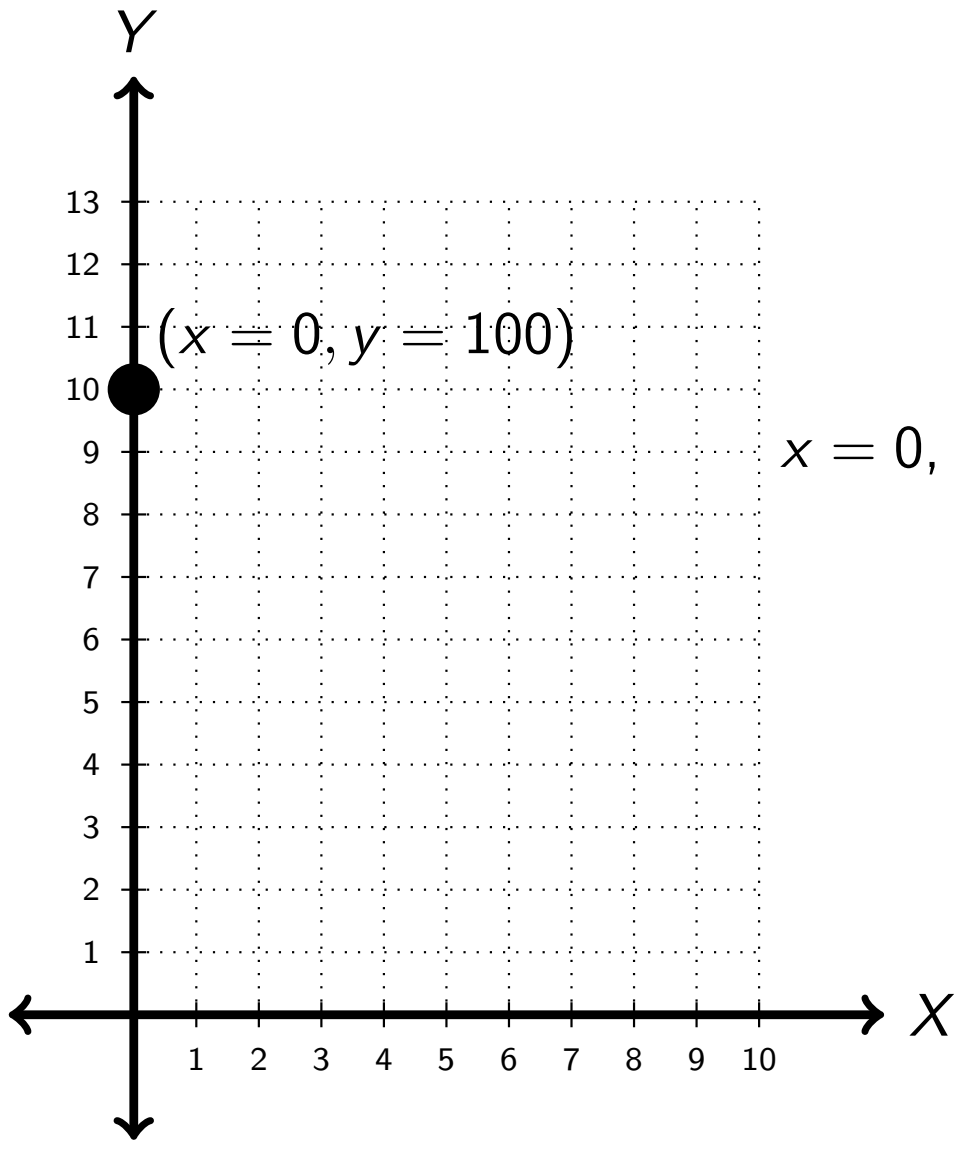
$$200x + 100y \leq 1000$$

# 3.1: Graphing inequalities




$200x + 100y = 1000$

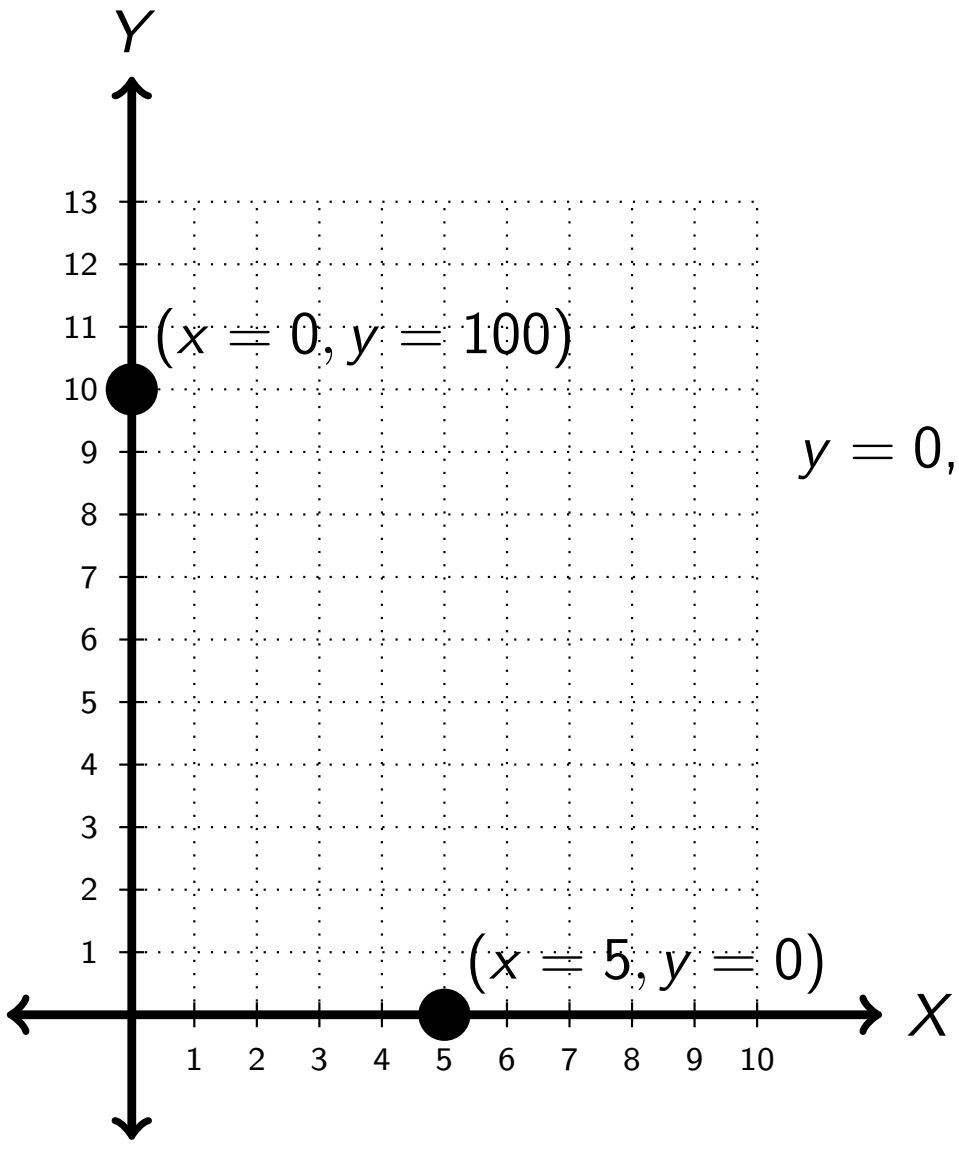
# 3.1: Graphing inequalities




$200x + 100y = 1000$

$x = 0,\ 100y = 1000,\ y = 10$

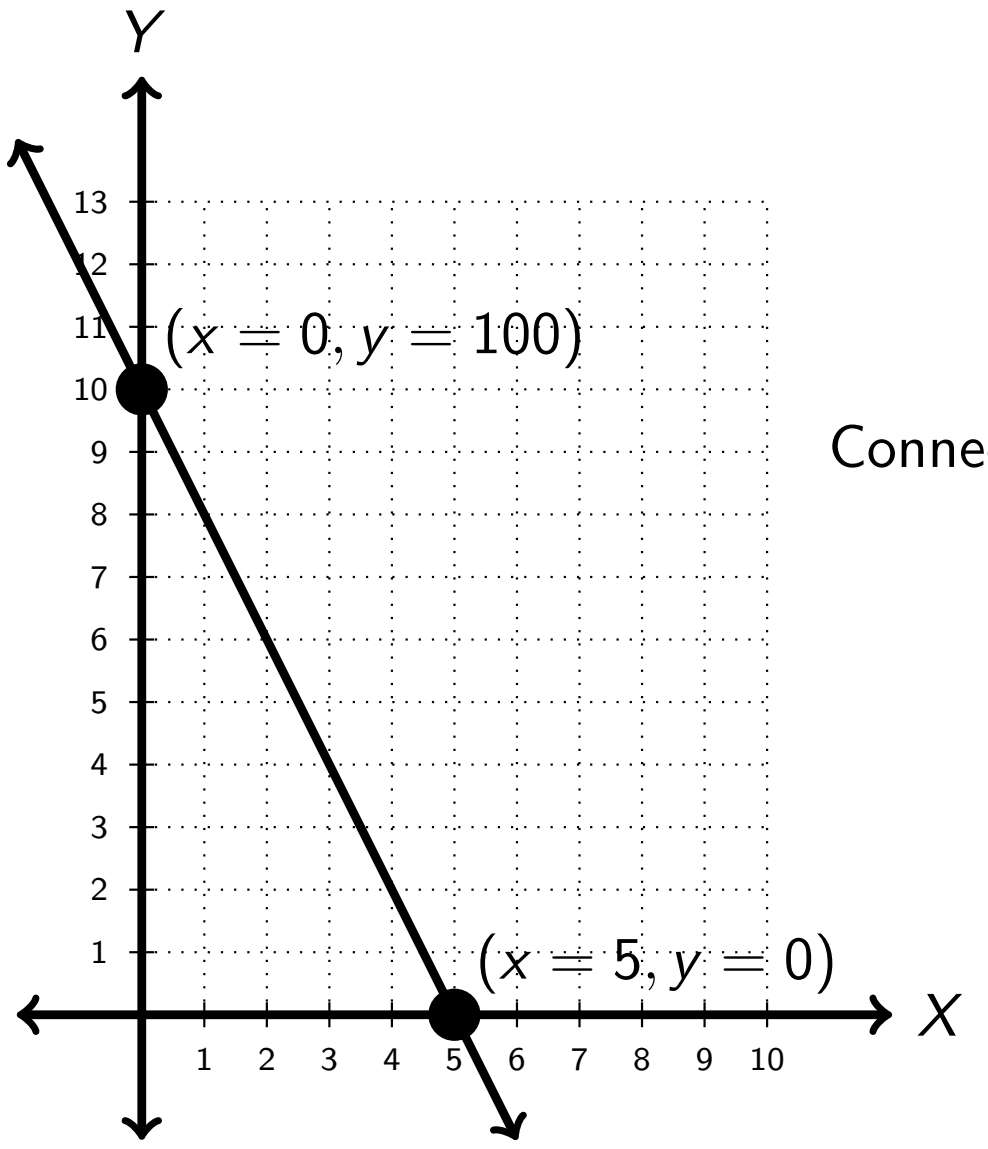
# 3.1: Graphing inequalities




$$200x + 100y = 1000$$

$$y = 0,\ 200x = 1000,\ x = 5$$

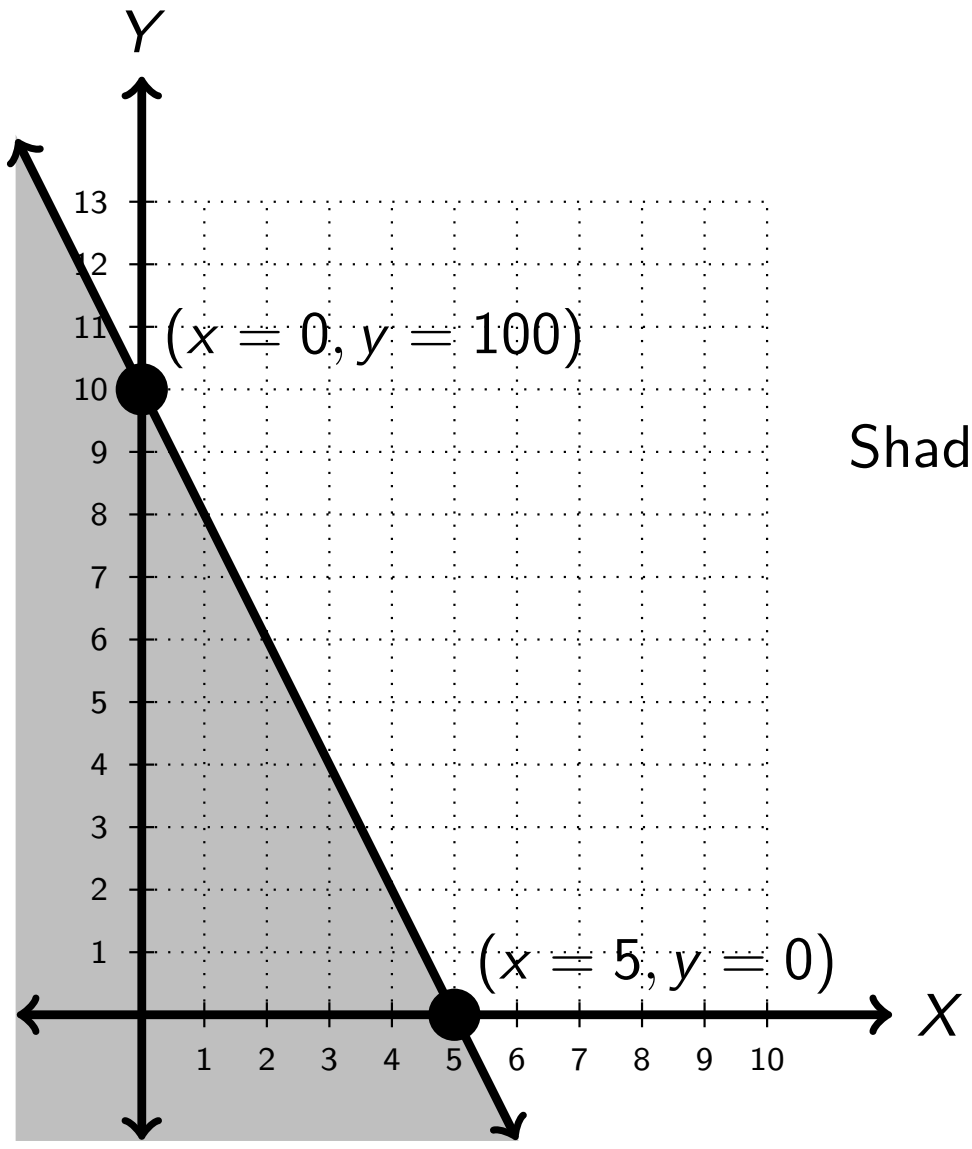
# 3.1: Graphing inequalities




$200x + 100y = 1000$

Connect the dots

# 3.1: Graphing inequalities




$200x + 100y \leq 1000$

Shade the region

# 3.1: Graphing inequalities

- First graph the "equality", that is, graph the line

$\Rightarrow$ Find two points on the line and then draw the connection

- Next graph the inequality, that is, shade the region

$\Rightarrow$ Choose a point not on the lines and see if it is on the correct side

- For example (0,0) is on the correct side since

$$(200)(0) + (100)(0) \leq 1000$$

# 3.1: Is it realistic?

- Our region is very large.
- Some points don't make sense for a single purchaser:

$\Rightarrow$ $(2.5, 3.5)$ means buy 2.5 Xylophones and 3.5 Yukuleles ($850)

- But maybe it makes sense as an average or a strategy
- Some points don't make any sense for any purchaser:

$\Rightarrow$ $(-10, -20)$ means buy -10 Xylophones . . . (-$4000)

# 3.1: Systems of inequalities

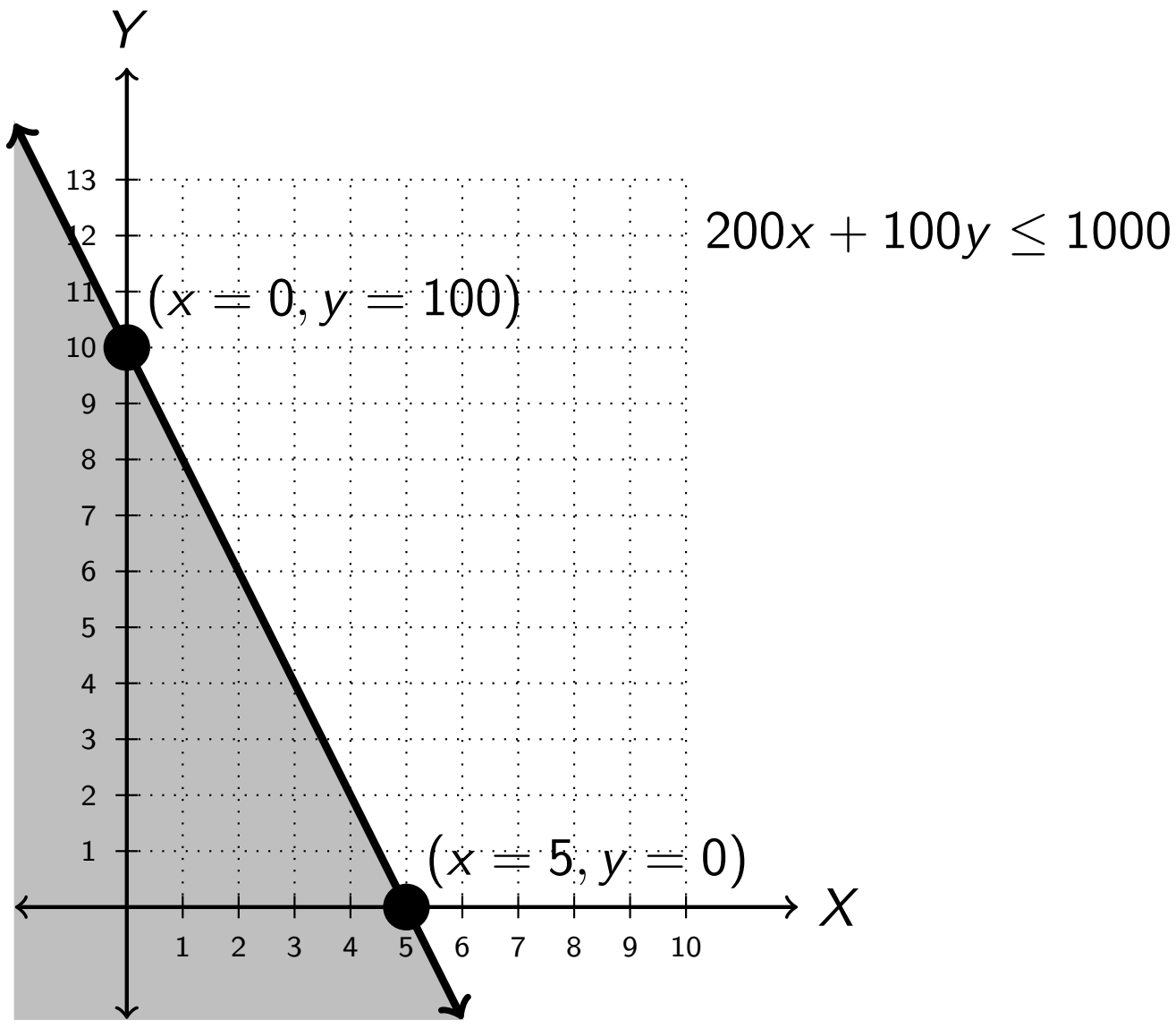
- We also need some sanity: $X \geq 0$ and $Y \geq 0$
- So we have a system of inequalities:

$$\begin{cases} 200X + 100Y \leq 1000 \\ X \geq 0, Y \geq 0 \end{cases}$$

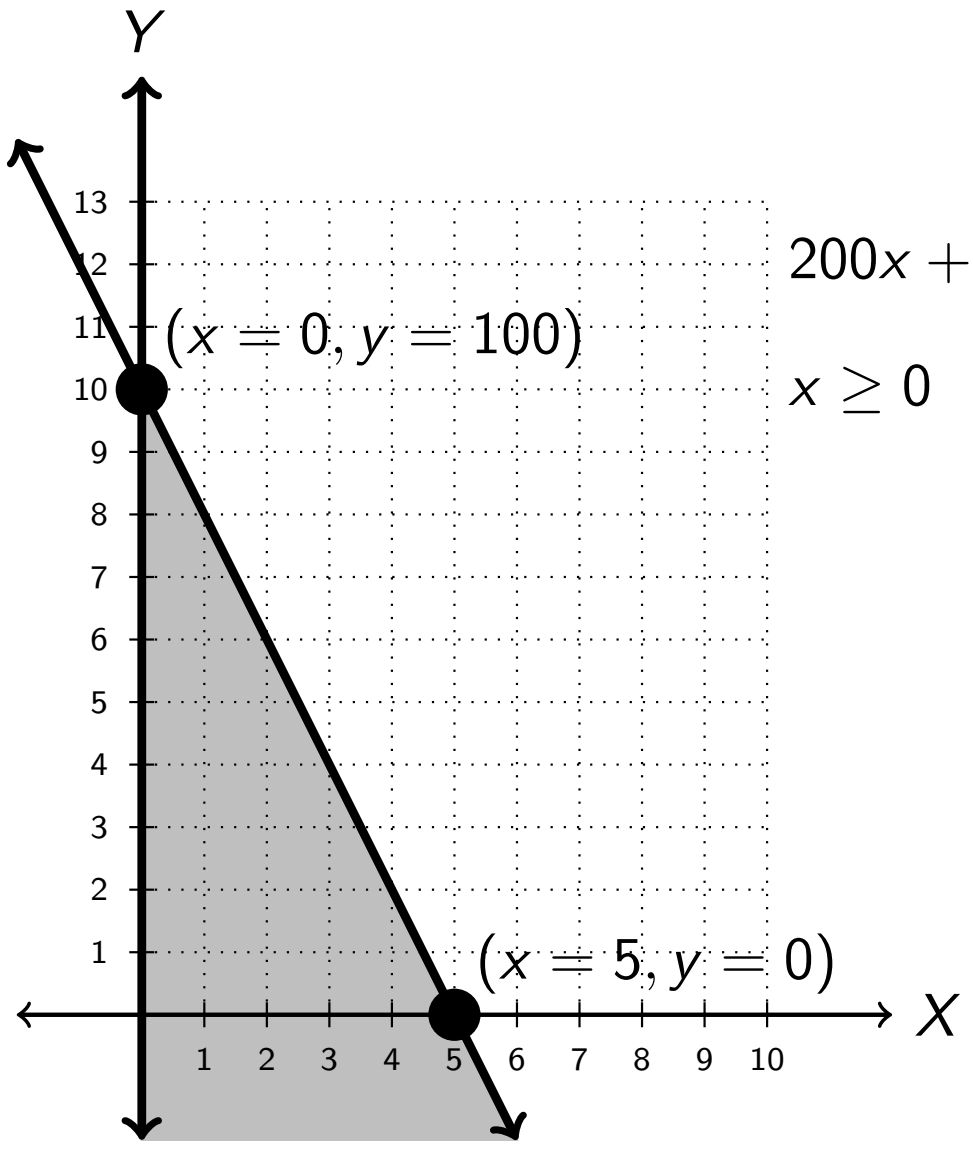
- Not enough for just one to be true!

$\Rightarrow$ $(500, 0)$ would be very expensive ($100,000) and noisy!

# 3.1: Graphing systems of inequalities



# 3.1: Graphing systems of inequalities



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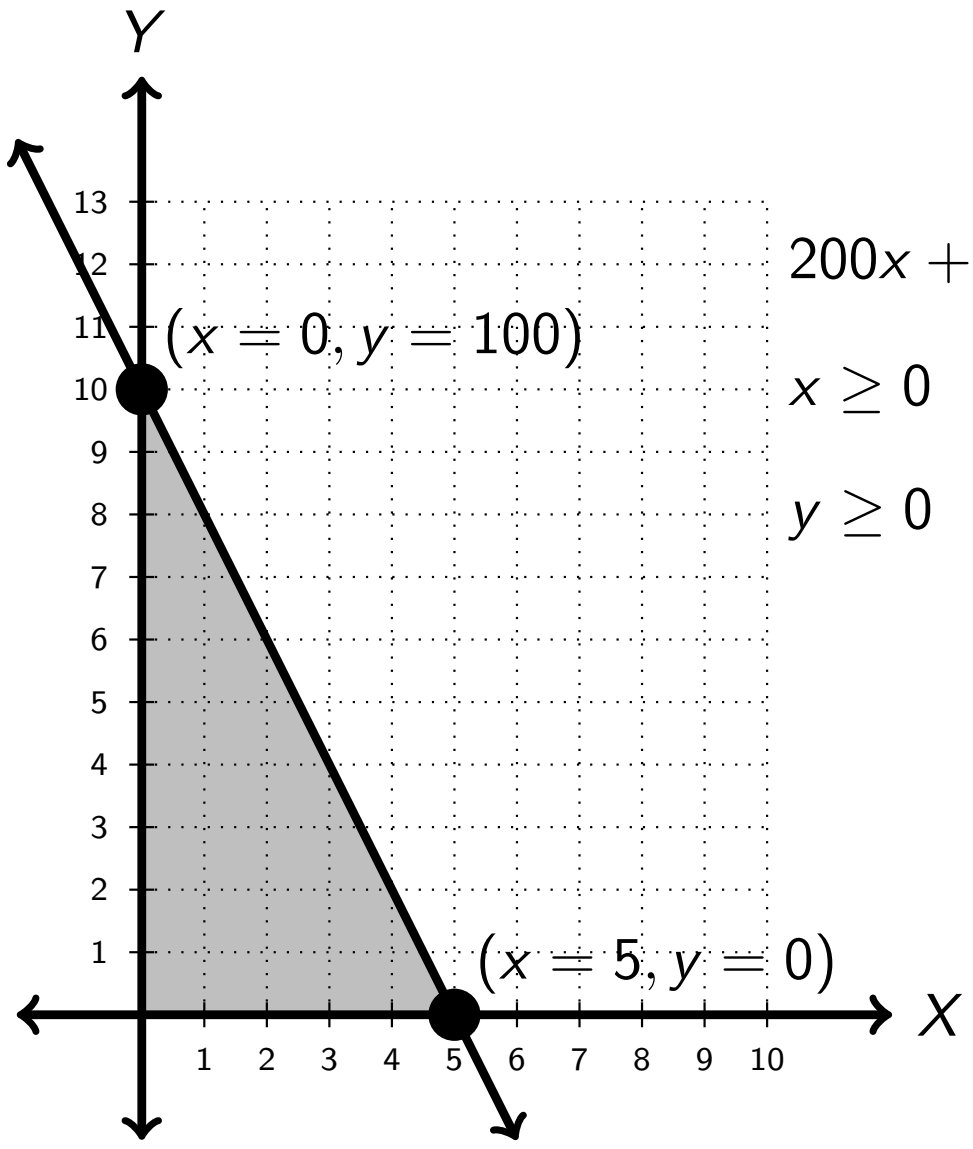



$200x + 100y \leq 1000$

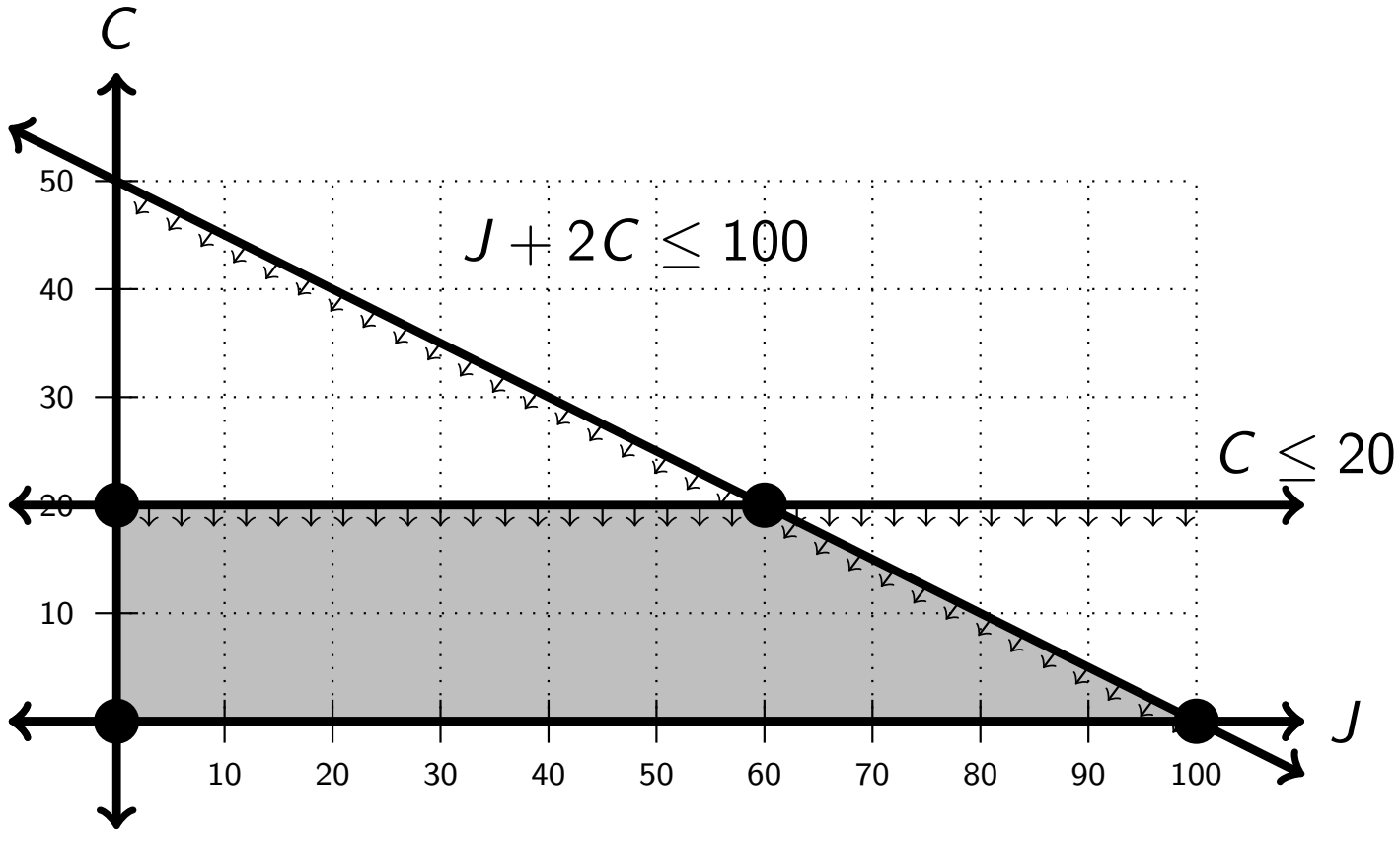
$x \geq 0$

$y \geq 0$

# 3.1: Graphing systems of inequalities

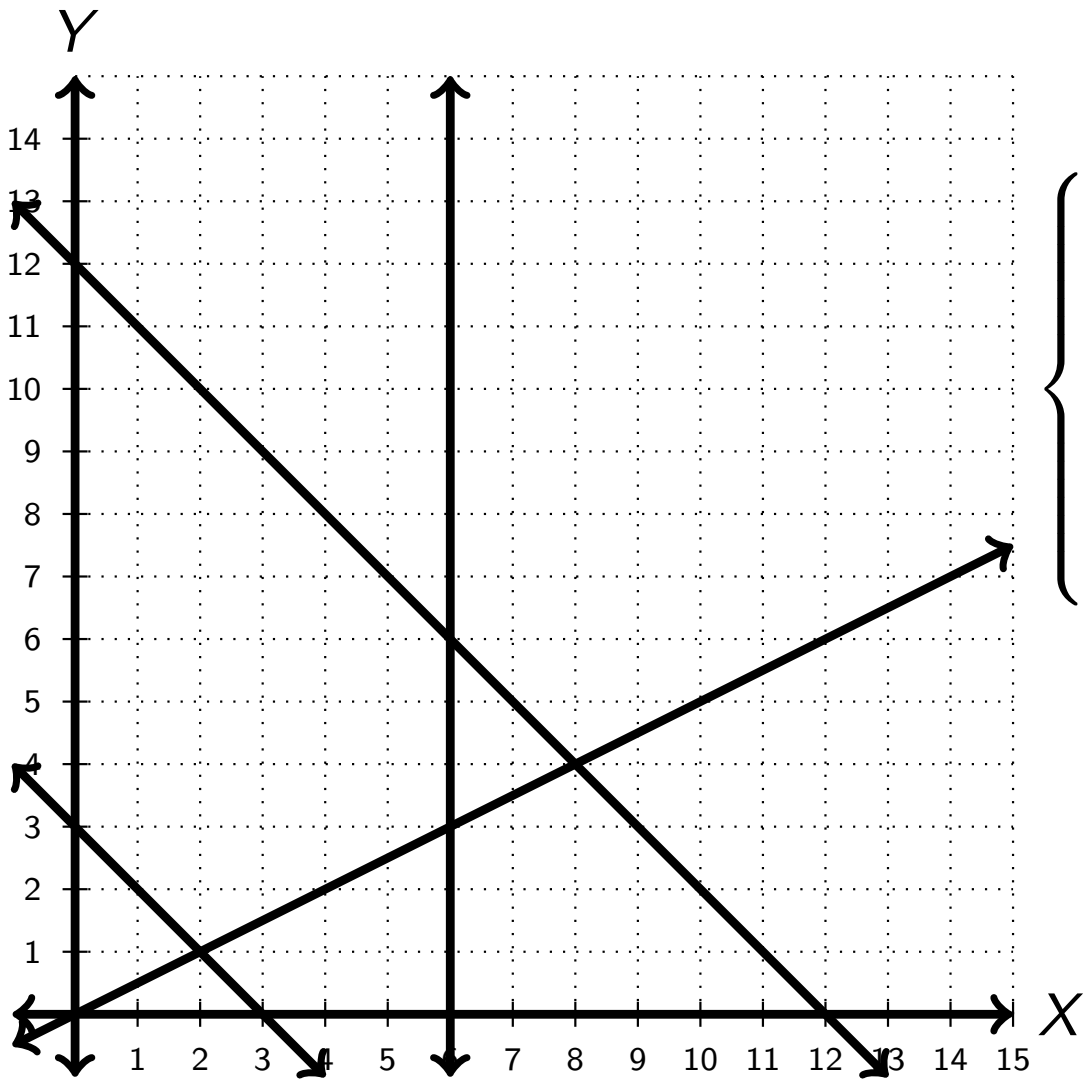
- Graph each equality (line)
- Figure out which side of the line is good
- Shade the region that is on the correct side of **all** lines
- Alternatively: figure out which of the pieces is good

# 3.1: Graphing systems of inequalities




Draw little arrows to show which side is good.

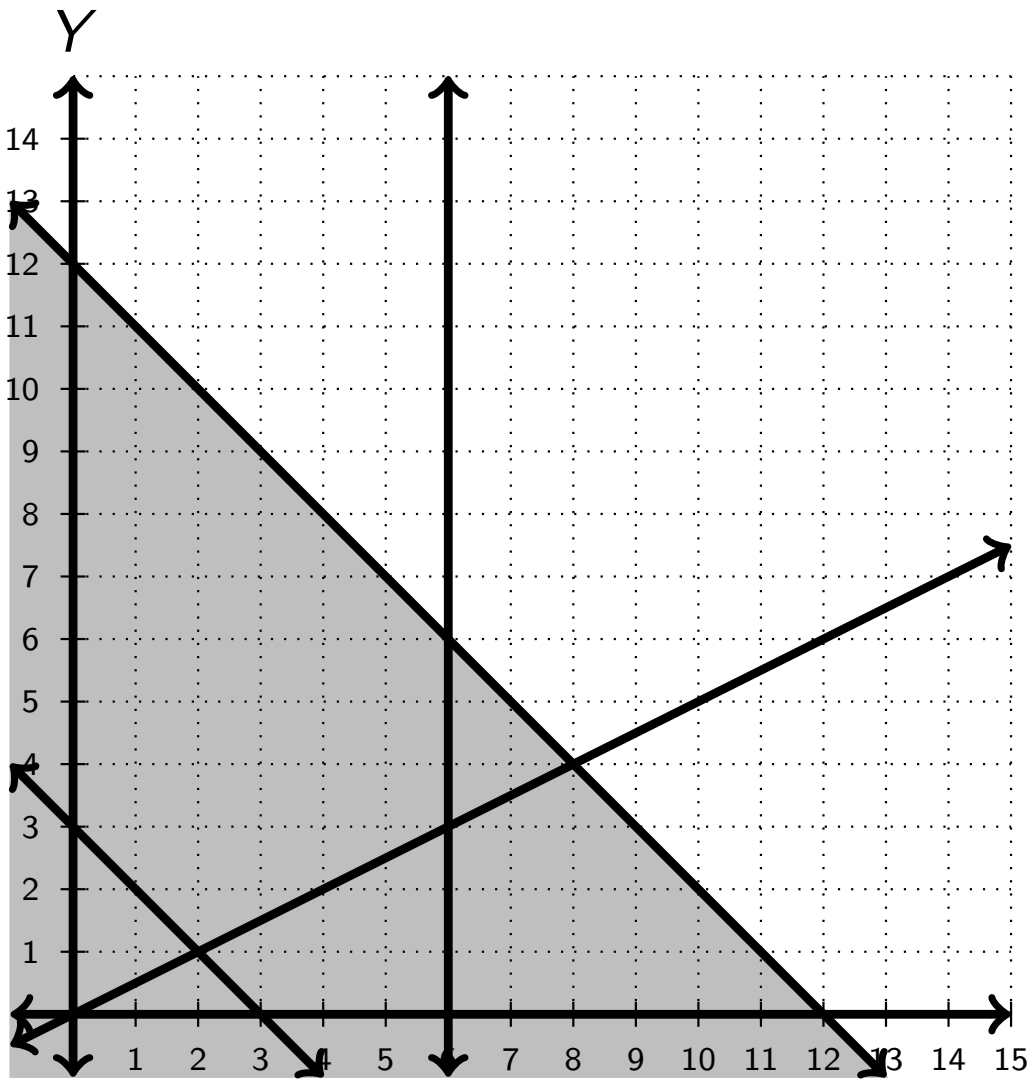
# 3.1: Graphing systems of inequalities




$$\begin{cases} x + y = 12 \\ x - 2y = 0 \\ x + y = 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

# 3.1: Graphing systems of inequalities

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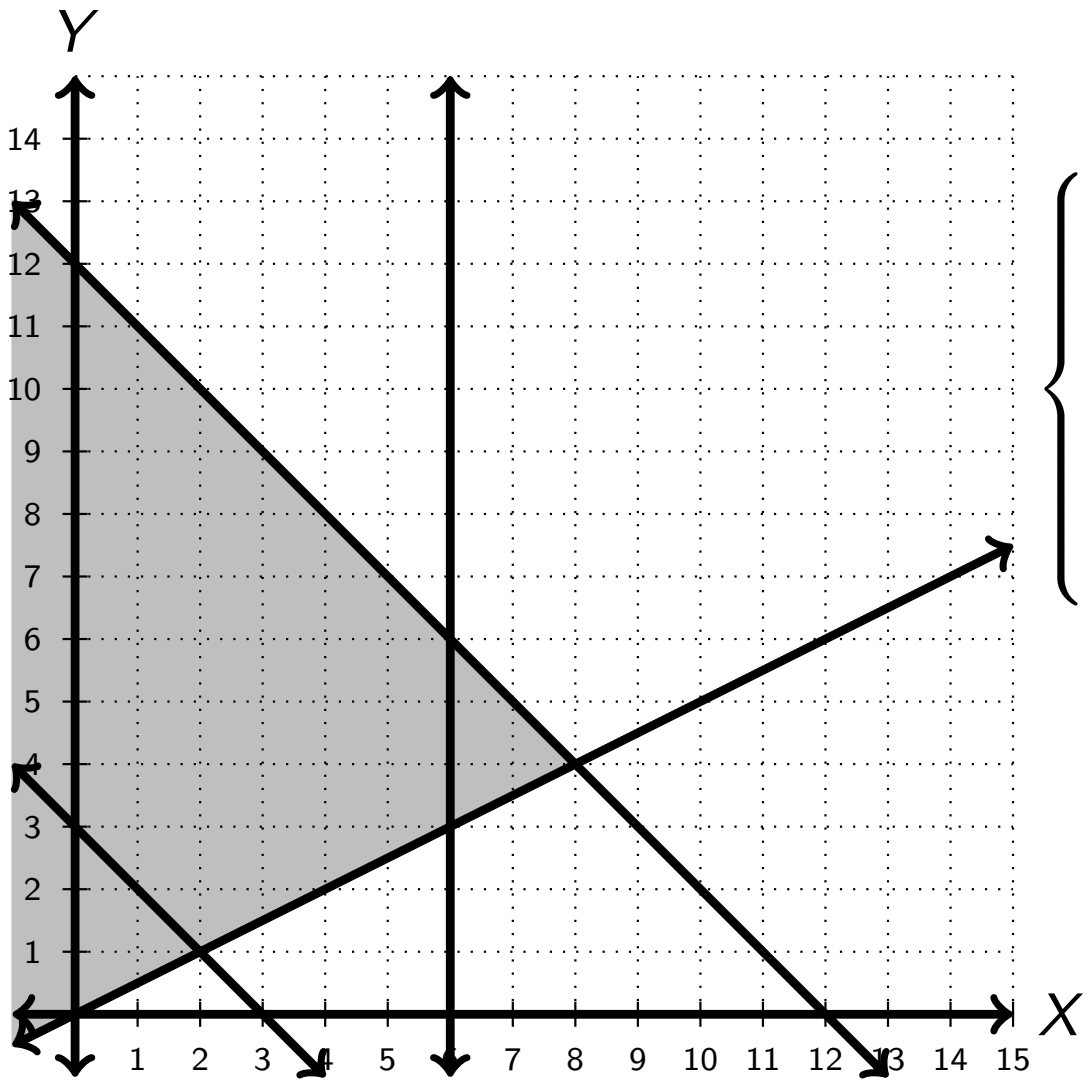


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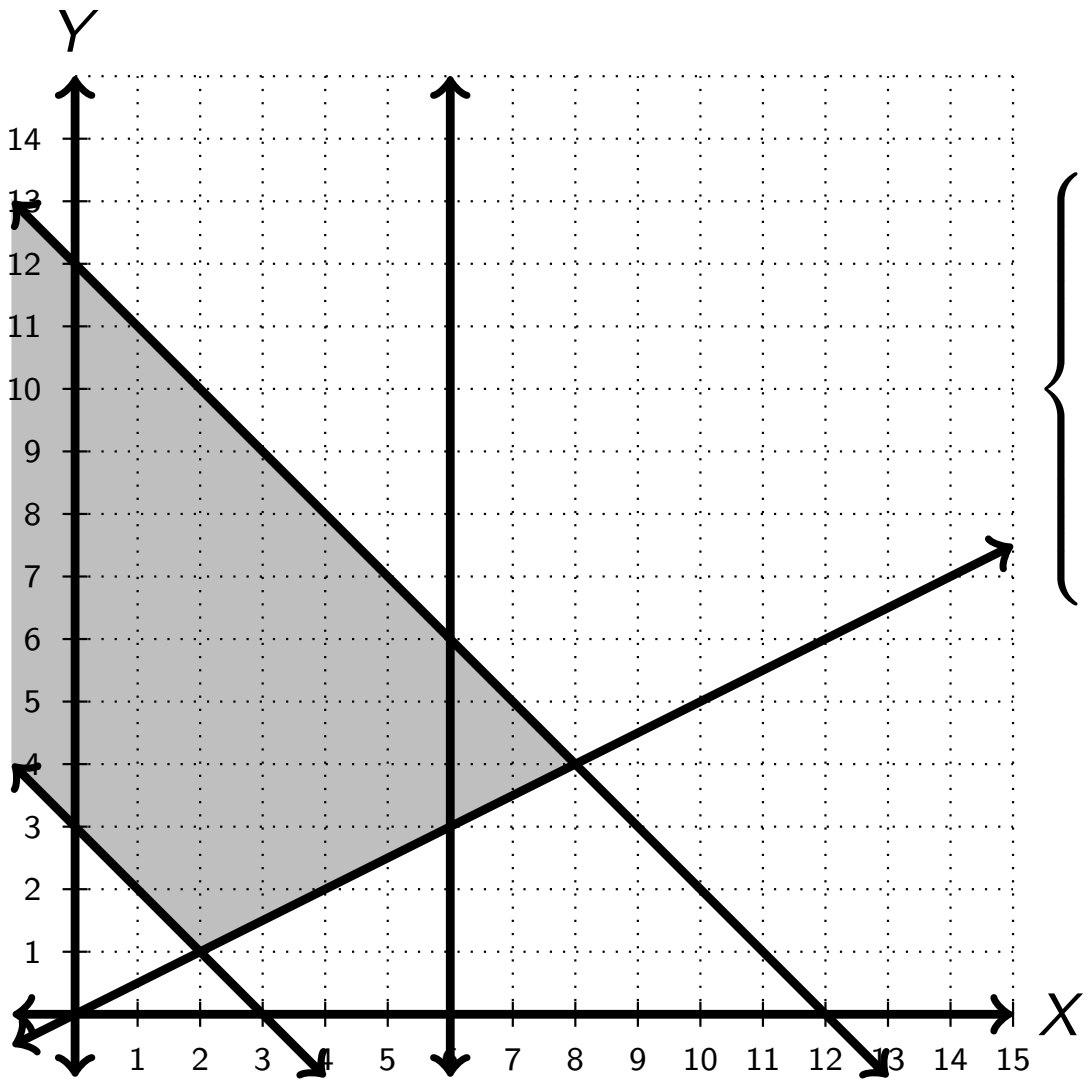


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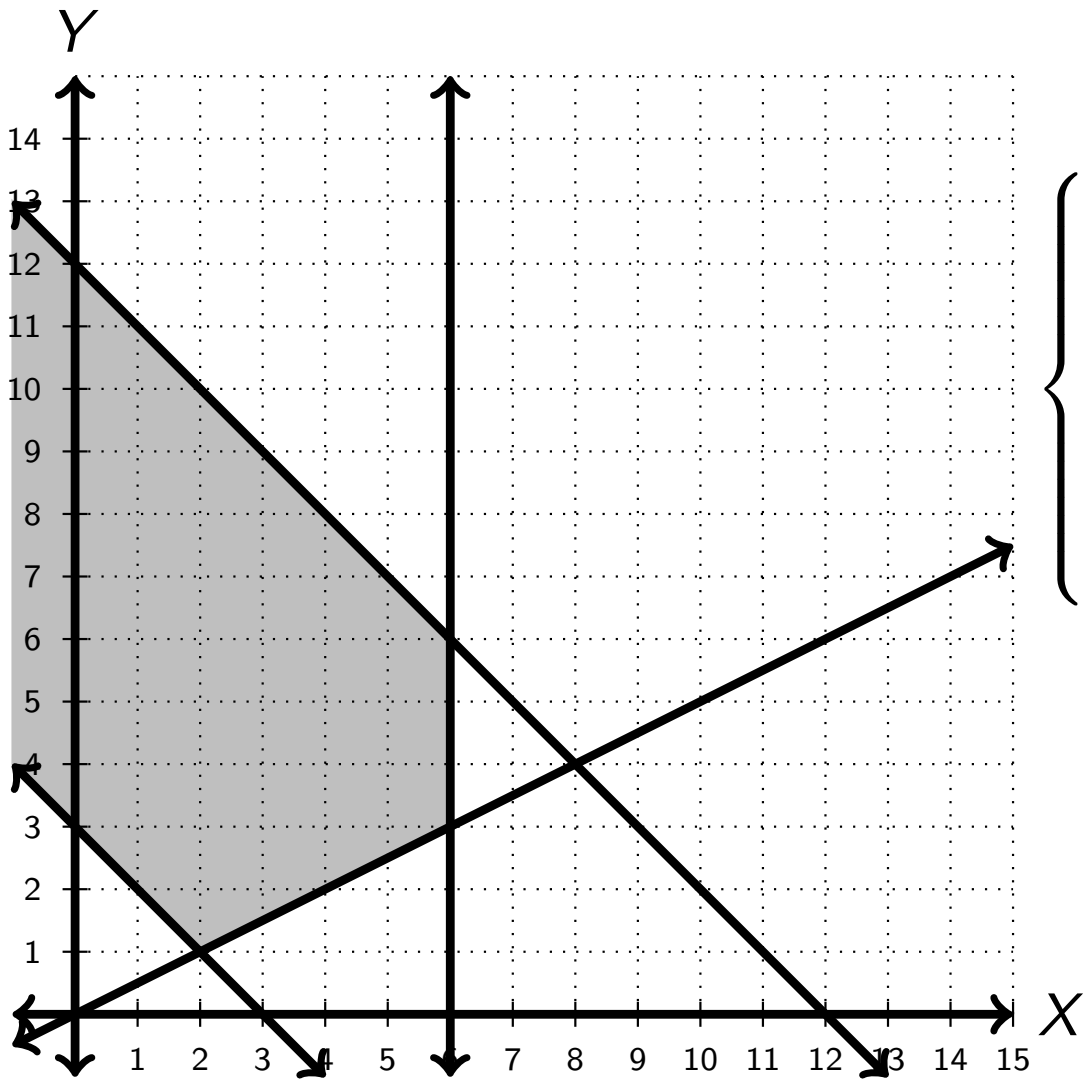


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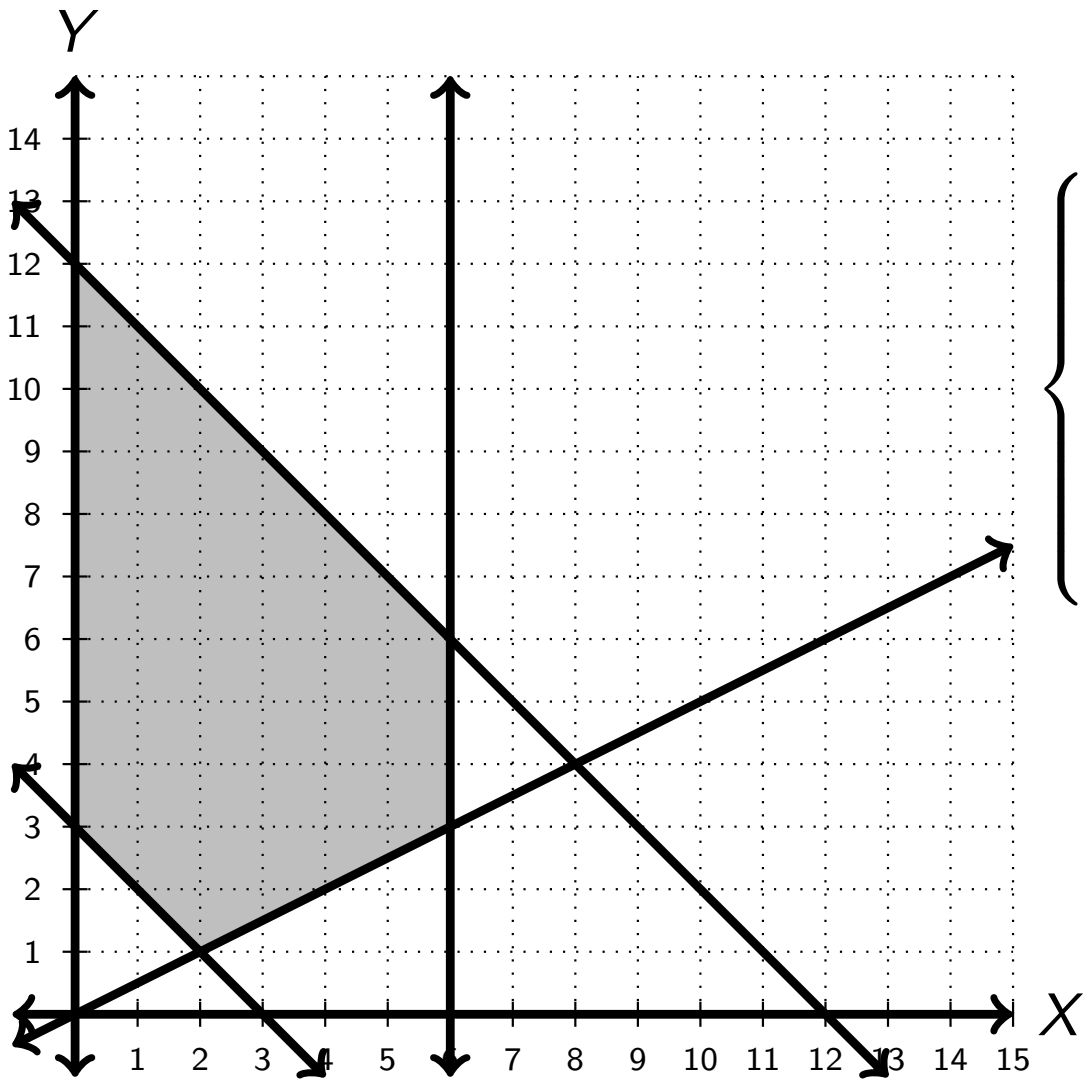


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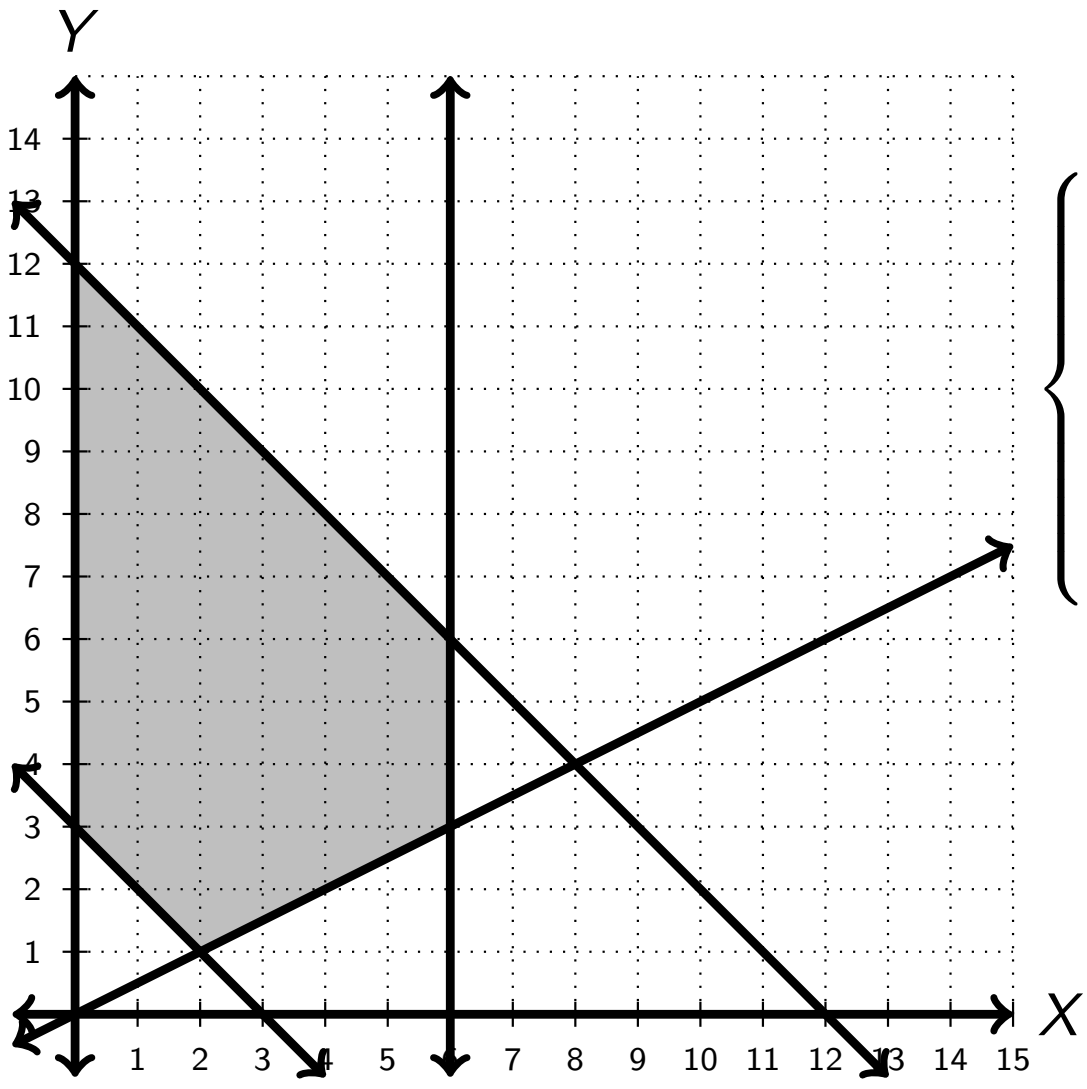
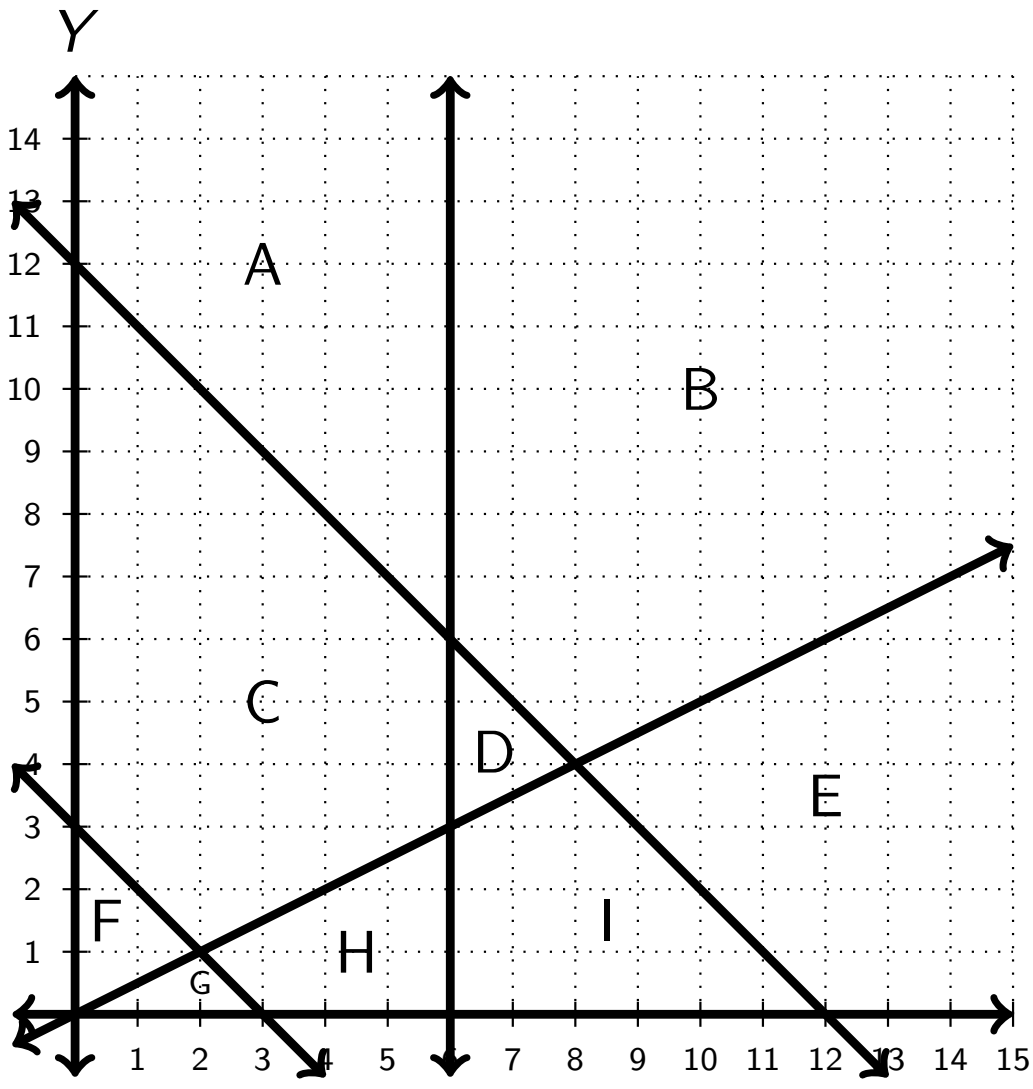



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Draw all the lines, then check each inequality. Too many regions!

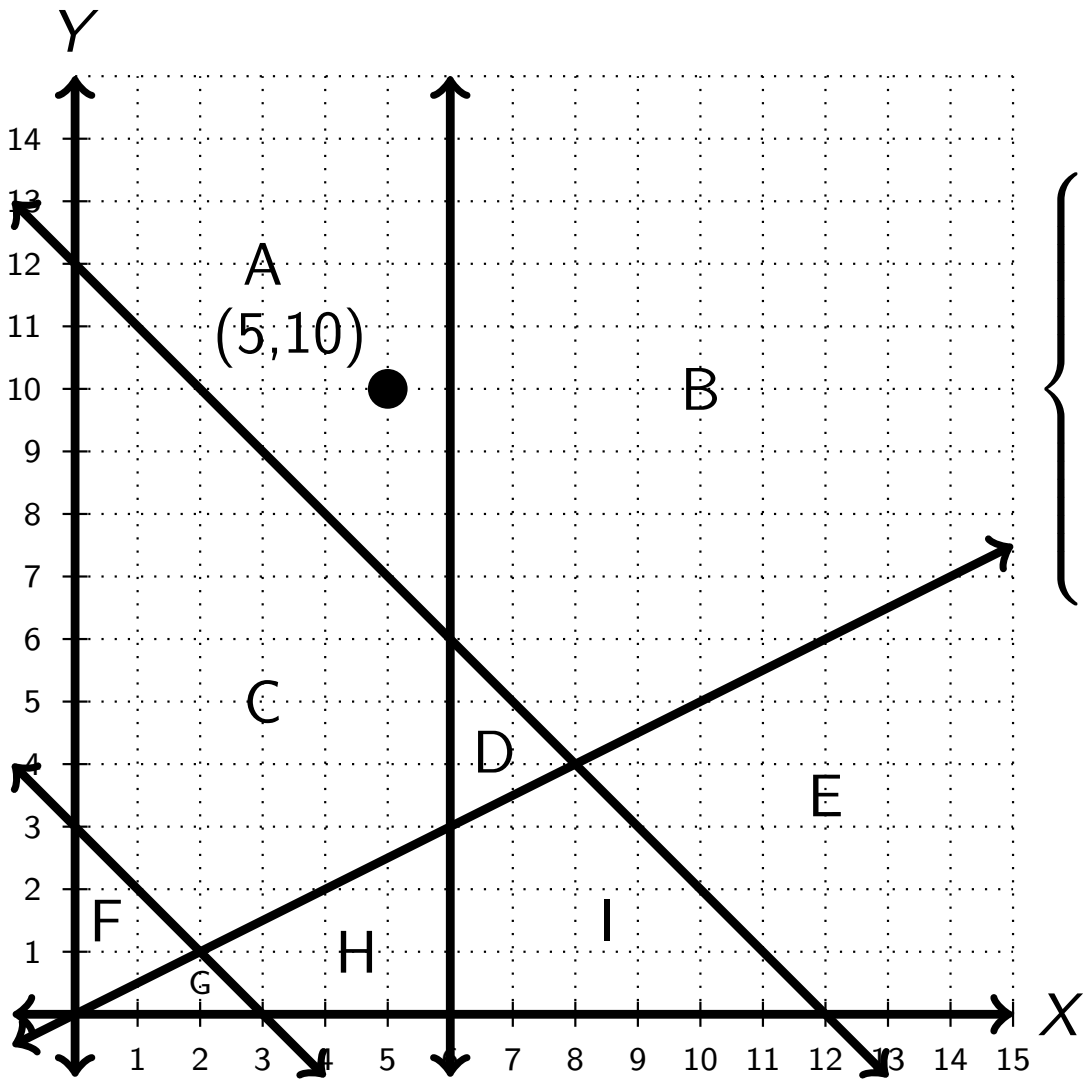
# 3.1: Graphing systems of inequalities




$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one.

# 3.1: Graphing systems of inequalities

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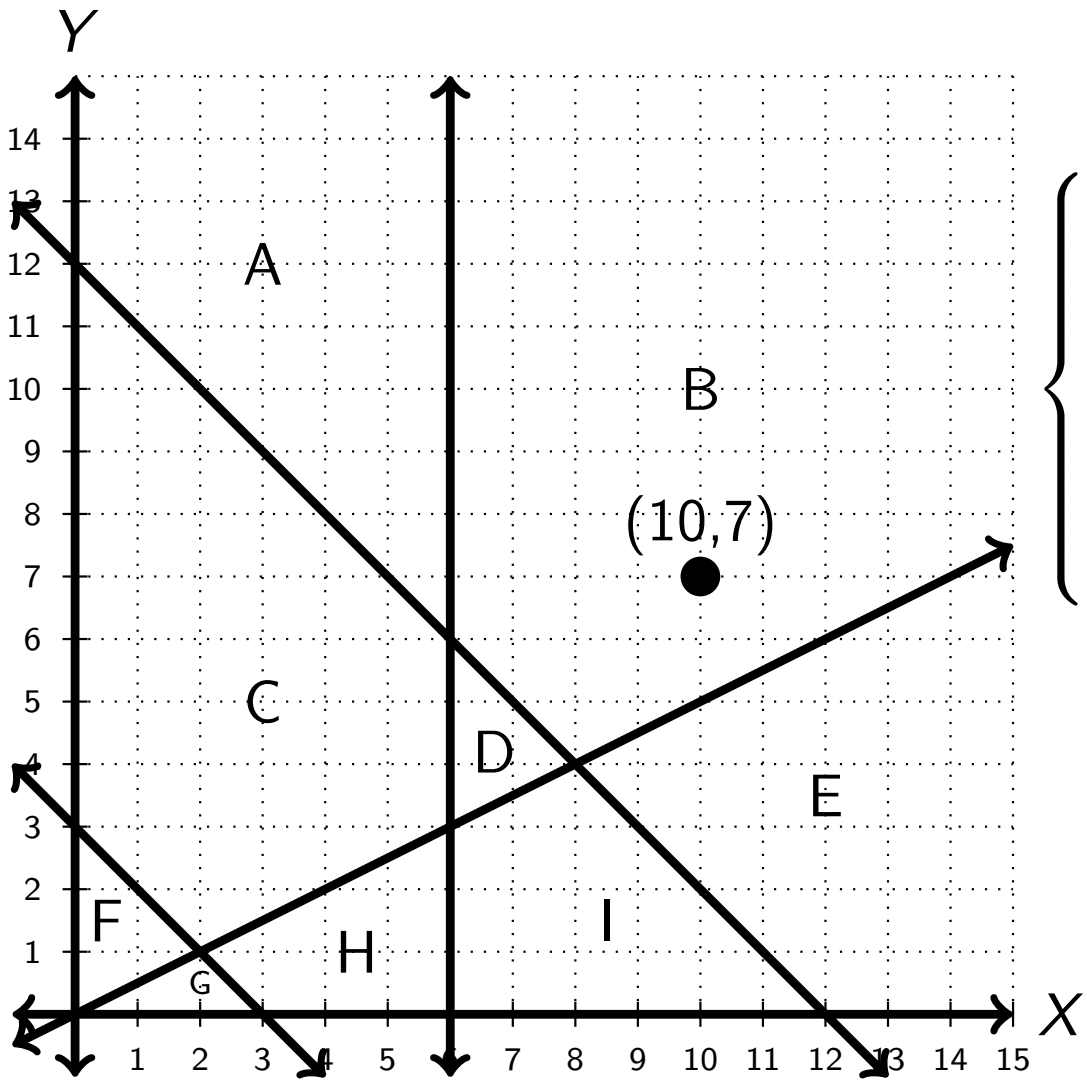


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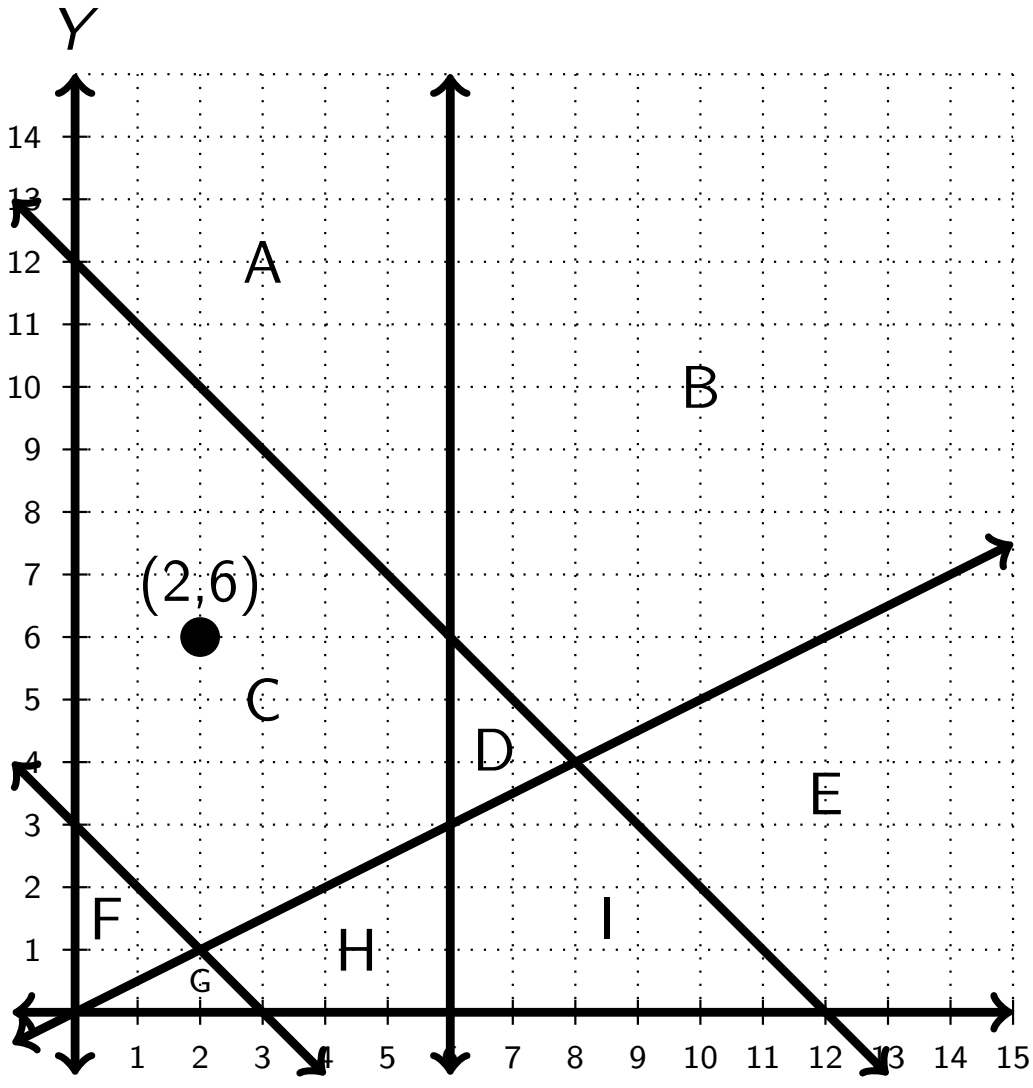


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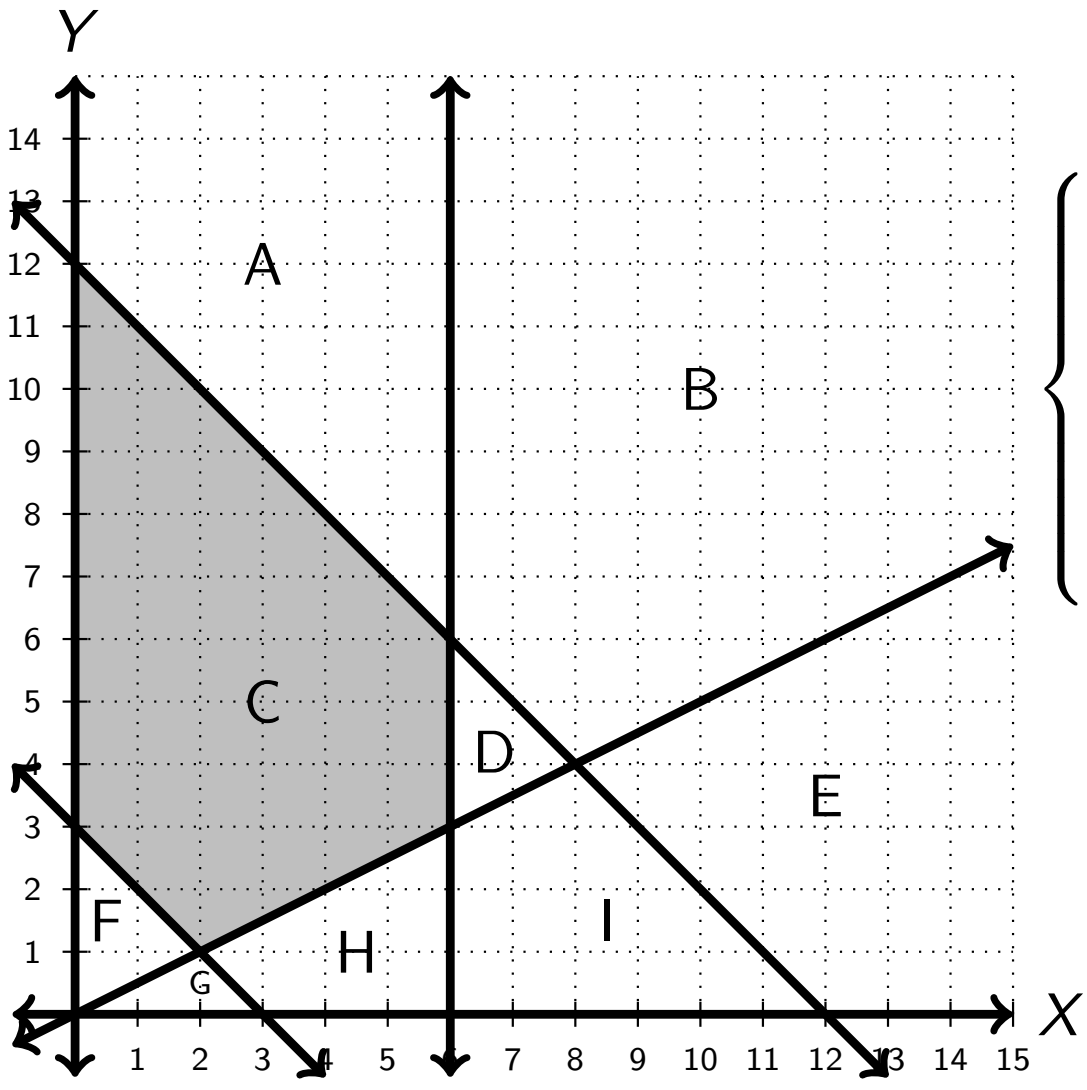
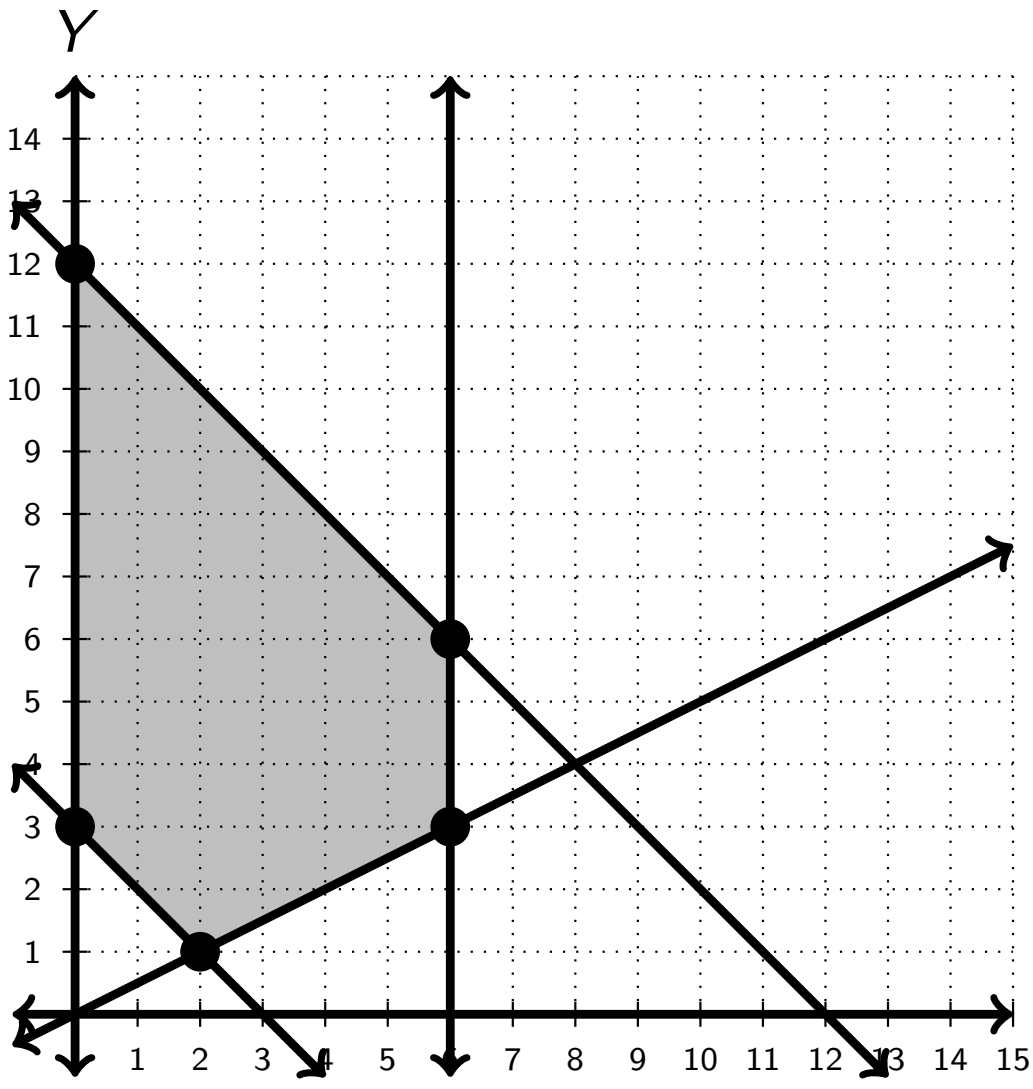



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one. Yay!

# 3.1: Finding corners




$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Intersect each pair of lines, and check it satisfies other inequalities

# 3.1: Finding corners

- For each pair of lines, find the intersection
- Then check that intersection satisfies the rest of the inequalities
- Not all intersections are corners!
- All corners are intersections.
- Intersections are just $2 \times 3$ RREF problems!