MA162: Finite mathematics

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SCHEDULE:

- HW C1 is due Monday, Mar 28th, 2011.
- HW C2 is due Monday, Apr 4th, 2011.
- HW C3 is due Sunday, Apr 10th, 2011.
- Exam 3 is Monday, Apr 11th, 5:00pm-7:00pm.
- No class Mar 14th, Mar 16th (Spring Break)

Today we will cover 5.1: simple and compound interest.
We will be using calculators today.

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money

    - Simple interest

    - Compound interest

    - Sinking funds

    - Amortized loans

- Chapter 6, Counting

    - Inclusion exclusion

    - Inclusion exclusion

    - Multiplication principle

    - Permutations and combinations

5.1: Interest

- Businesses often need short-term use of expensive assets, so find renting attractive (often tax-deductible)

- Sometimes what a business needs most is just cash. In a small business, you don't make money every day. A successful small business does make money, so can repay the money in the future.

- How can they rent cash? Why would somebody give them money today? For the promise of more money in the future. **Interest**

- How much more?

    - The more money being loaned, the more interest. **Principal**

    - The longer the money is loaned, the more interest. **Time**

5.1: Simple interest

- For short term loans, people use a **simple** model for interest

$$I = Prt$$

- There is the **Principal**, the amount of money borrowed, like \$100

- There is a **rate** of interest, like 10% per year

- There is a **time** period, after which the money is due, like 1 year

- There is the **Interest**, the extra money due at the end,

  like $(\$100) \cdot (10\% \text{ per year}) \cdot (1 \text{ year}) = \$10$.

5.1: Simple interest examples

$$I = Prt$$

- If $100 is lent at 10% interest per year for six months, then

$$I = (\$100) \cdot (10\% \text{ per year}) \cdot (\tfrac{1}{2} \text{ year}) = \$5$$

- If $100 is lent at 7% interest per year for three months, then

$$I = (\$100) \cdot (7\% \text{ per year}) \cdot (\tfrac{1}{4} \text{ year}) = \$1.75$$

- If $325 is lent at 12% interest per year for five months, then

$$I = (\$325) \cdot (12\% \text{ per year}) \cdot (\tfrac{5}{12} \text{ year}) = \$16.25$$

5.1: More examples

- What is the simple (yearly) interest rate if \$100 is loaned for 3 months with \$5 interest due?

$P = \$100$	$I = Prt$
$t = 1/4$ year	$5 = (100)(r)(1/4)$
$I = \$5$	$5 = 25r$
$r = ?$	$r = 1/5 = 20\%$

- If the interest rate is 7% and \$9.10 interest is due after three months, how much was loaned?

$r = 7\%$ per year	$I = Prt$
$t = 1/4$ year	$\$9.1 = P(7\%)(1/4)$
$I = \$9.10$	$\$36.4 = P(7\%)$
$P = ?$	$P = \$36.4/7\% = \$520$

5.1: Simple interest as rent

- Simple interest treats loaned money like a loaned house.

- Every month you pay money to borrow the house, and at the end of the year you give the house back.

- The owner goes without his house for a year, but receives money in exchange (rent). The lender goes without his money for a year, but receives money in exchange (interest).

- How much rent do you pay total? If each month you pay \$300, then by the end of the year, you've paid $(12)(\$300) = \$3600$.

- This calculation is similar to the interest calculations we just did. Each month you pay $(\$100) \cdot (12\% \text{ per year})(\frac{1}{12} \text{ year}) = \$1$ interest, and at the end of the year that is $(12)(\$1) = \$12$, or 12% of the original loan.

5.1: Got no money to pay the rent

- What happens when you cannot pay the rent?
  Well, typically lots of bad things.

- What if you borrow the rent from your friend?
  Maybe he charges interest too.

- What if you can't pay back your friend?
  Maybe you borrow from another friend,
  and maybe they charge interest too.

- Interest on interest is called **compound interest**

- In finance and economics, nearly all interest is compound

- Simple interest is used for short-term loans

5.1: Three Brothers Loans and Cement Mixing, LLC

- Harry, Gary, and Scary run a limitedly legal company specializing in short term loans

- In January, Bob borrowed $100 from Harry for one month at 10% per **month** interest

- In February, Gary stopped by to say hello, and that his brother was anxious for his $110

- Bob didn't have the $110, but Gary said he looked like a nice guy and would loan him the $110 at 10% per month interest.

- Bob asked if Gary minded giving Harry the money, since they were brothers, and so Gary took back the money immediately and went back to the cement yard.

5.1: Three Brothers Loans and Cement Mixing, LLC

- In March, Scary stopped by to say hello, and that his brother was anxious for his $121

- Bob didn't have the $120, wait, $121?

- If Bob had borrowed $100 for 2 months at 10% per month interest, then he would owe:

  $100 + ($100) \cdot (10\% \text{ per month}) \cdot (2 \text{ months}) = \$100 + \$20$

  However, he had borrowed $110 from Gary for 1 month at 10% per month interest, so he owed:

  $110 + ($110) \cdot (10\% \text{ per month}) \cdot (1 \text{ month}) = \$110 + \$11$

- Scary had no interest in the math, only in the interest, $10 from the first month, $11 from the second

5.1: Compound interest

- The most basic formula for compound interest is:

$$A = P(1 + i)^n$$

- the **Principal** is the amount initially borrowed, like \$100

- the **interest rate** per compounding period, like 10% per month

- the **number** of compounding periods that have passed, like 2 months

- the **Accumulated Amount** of money due, both the principal and the interest, like

$$(\$100)(1 + 10\%)^2 = (\$100)(1.10)^2 = \$121$$

5.1: Compound interest examples

$$A = P(1 + i)^n$$

- If you borrow \$100 at 10% per month, compounded monthly, for six months you owe

$$(\$100) \cdot (1.1)^6 \approx \$177.16$$

- If you borrow \$100 at 10% per month, compounded monthly, for nine months you owe

$$(\$100) \cdot (1.1)^9 \approx \$235.79$$

- If you borrow \$100 at 10% per month, compounded monthly, for twelve months you owe

$$(\$100) \cdot (1.1)^{12} \approx \$313.84$$

5.1: Why does the formula work?

- If you borrow \$100 at 10% per month, compounded monthly, for one month you owe

$$\$100 + (\$100)(10\%) = (\$100) \cdot (1 + 10\%) = (\$100) \cdot (1.1) = \$110$$

- If you borrow it for another month, you owe

$$\$110 + (\$110)(10\%) = (\$110)(1 + 10\%) = (\$110)(1.1)$$

$$= (\$100)(1.1)(1.1) = (\$100)(1.1)^2 = \$121$$

- If you borrow it for another month, you owe

$$\$121 + (\$121)(10\%) = (\$121)(1 + 10\%) = (\$121)(1.1)$$

$$= (\$100)(1.1)^2(1.1) = (\$100)(1.1)^3 = \$133.10$$

5.1: Confusing customers for fun and profit

- Stating interests rates in terms of months, fortnights, or furlongs makes it hard to compare interest rates

- A simple way to handle this is to multiply the rate by how many periods there are per year, to "convert" to a yearly rate, like $(10\% \text{ per month}) \cdot (12 \text{ months per year}) = 120\%$ per year

- The **nominal rate** is this rate, "120% interest per year, compounded monthly"

- To convert from a nominal rate to a per-period rate just divide by the number of periods

- a nominal rate of 12% per year compounded monthly is a rate of $(12\% \text{ per year})/(12 \text{ months per year}) = 1\%$ per month

5.1: Confusing customers for fun and profit

- However, what happens to Bob (best-case scenario) if he continues to get loans from the three brothers?

- The nominal rate was 120% per year, compounded monthly

Jan	Feb	Mar	Apr	May	Jun
$100	$110	$121	$133.10	$146.41	$161.05

Jul	Aug	Sep	Oct	Nov	Dec
$177.16	$194.87	$214.36	$235.79	$259.37	$285.31

Jan
$313.84 $= \$100 +$ $213.84

- "120% per year compounded monthly" fails to capture the "213.84% per year simple interest"

5.1: Effective interest rate

- In the U.S. the 1968 Truth in Lending Act required lenders to advertise the **effective** annual percentage rate

- The true calculation is complicated, depends on your jurisdiction, and takes into account certain fees and penalties.

- In MA162, the formula is not so complicated. You just calculate the interest for one year.

- For instance, the three brothers nominal rate of 120% resulted in

$$(1+\tfrac{1.20}{12})^{12}-1 = (1+0.10)^{12}-1 = 1.1^{12}-1 \approx 2.13843 = 213.843\%$$

- In general

$$r_{eff} = (1 + \tfrac{r}{m})^m$$

5.1: Summary

- Today we learned **simple interest**, **compound interest**, and the **effective interest rate**.

- We used the words **interest**, **principal**, **interest rate**, **compounding period**, **nominal rate**, **accumulated amount**.

- You are now ready to complete HWC1 #s 1,5,6,7,8,9,10,12,13

- Make sure to take advantage of office hours, and have your questions ready for your next recitation