MA162: Finite mathematics

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March 28, 2011

Schedule:

- HW C1 is due Today, Mar 28th, 2011.
- HW C2 is due Monday, Apr 4th, 2011.
- HW C3 is due Sunday, Apr 10th, 2011.
- Exam 3 is Monday, Apr 11th, 5:00pm-7:00pm.
- Alternate exam form (again) due Thursday, Mar 31, 2011.

Today we will cover 6.1: Sets

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations

6.1: Life before sets

- We are going to be doing some hard counting problems.
- To make it easier, we need to be able to talk about the things we are counting.
- When we counted money, or acres, or ounces of jamba juice we had variables to denote the number. $x = 5$ acres, or $y = 10$ ounces.
- If you had \$5 in one bank account and \$10 in another, you had \$5+\$10 = \$15 total. The numbers were all that mattered.
- Unfortunately life rarely divides nicely into separate accounts, and numbers cannot describe many of these aspects.

6.1: More than numbers can say

- We are going to be counting more complicated things now.
- If your friend Jimmy says you can borrow their car Monday, Tuesday, and Wednesday, then that is 3 days you've got a car.
- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?

6.1: More than numbers can say

- We are going to be counting more complicated things now.
- If your friend Jimmy says you can borrow their car Monday, Tuesday, and Wednesday, then that is 3 days you've got a car.
- If your friend Timmy says you can borrow their car Tuesday, Thursday, and Friday, then that is 3 days you've got a car.
- How many days total can you borrow a car?
- Well, Monday, Tuesday, Wednesday, Thursday, Friday is five days.
- But $5 \neq 3 + 3$. Numbers are not enough.

6.1: Sets to name the things we are counting

- If we let J be the days Jimmy lets us have the car, then

$$J = \{ \text{ Monday, Tuesday, Wednesday } \}$$

- If we let T be the days Timmy lets us have the car, then

$$T = \{ \text{ Tuesday, Thursday, Friday } \}$$

- The days when at least one of them let us use the car is the **union** of the two sets

$$J \cup T = \{ \text{ Monday, Tuesday, Wednesday, Thursday, Friday } \}$$

- The days when both of them let use the car is the **intersection** of the two sets

$$J \cap T = \{ \text{ Tuesday } \}$$

6.1: More sets

- We can have sets of numbers $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then:
- $A \cup B = \{1, 2, 3, 4, 5\}$
- $A \cap B = \{3\}$
- $A - B = \{1, 2\}$ is the difference, the things in A that are not in B
- We can write down sets in funny ways:
  $A = \{3, 2, 1\} = \{1, 1, 1, 1, 1, 2, 2, 3\}$
- We can describe them in words, "A is the set of positive integers whose square is a one digit number."

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 3\}$
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- $\{1, 2, 3\} \stackrel{?}{=} \{1, 2, 2, 3, 3, 3\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ positive integers whose square has one digit $\}$
- $\{1, 2, 3\} \stackrel{?}{=} \{$ odd numbers less than 4 $\}$

6.1: Equality drill

- Two sets are **equal** if they have the same elements.
- $\{1, 2, 3\} = \{1, 2, 3\}$
  Yes! Exactly the same.
- $\{1, 2, 3\} \neq \{1, 2\}$
  No! Right hand set is missing "3"
- $\{1, 2, 3\} = \{3, 1, 2\}$
  Yes! Order does not matter.
- $\{1, 2, 3\} = \{1, 2, 2, 3, 3, 3\}$
  Yes! Repeats don't matter.
- $\{1, 2, 3\} = \{$ positive integers whose square has one digit $\}$
  Yes! Long-winded doesn't matter.
- $\{1, 2, 3\} \neq \{$ odd numbers less than 4 $\}$
  No! Right hand set is missing "2"

6.1: Union and intersection drill

- $\bigcup$ The **union** includes anything in either, and is big. $\bigcup$
- $\bigcap$ The **intersection** includes only those in both, and is small. $\bigcap$
- $\{1, 2, 3\} \cup \{3, 4, 5\} =$
- $\{1, 2, 3\} \cap \{3, 4, 5\} =$
- $\{1, 2, 3\} \cup \{1\} =$
- $\{1, 2, 3\} \cap \{1\} =$

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- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
- $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
- $\{1, 2, 3\} \cup \{1\} = \{1, 2, 3\}$
- $\{1, 2, 3\} \cap \{1\} = \{1\}$

6.1: Difference drill

- $-$ The **difference** keeps the first, but not in the second. $-$
- $\{1, 2, 3\} - \{1\} =$
- $\{1, 2, 3\} - \{2, 3\} =$
- $\{1, 2, 3\} - \{3, 4, 5\} =$
- $\{1, 2, 3\} - \{4, 5, 6\} =$
- $\{1, 2, 3\} - \{1, 2, 3\} =$

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- $\{1, 2, 3\} - \{2, 3\} = \{1\}$
- $\{1, 2, 3\} - \{3, 4, 5\} = \{1, 2\}$
- $\{1, 2, 3\} - \{4, 5, 6\} = \{1, 2, 3\}$
- $\{1, 2, 3\} - \{1, 2, 3\} = \{\}$ The **empty set** containing nothing.

6.1: Laws of sets

- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$, but what about $\{3, 4, 5\} \cup \{1, 2, 3\}$?

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- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$,
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- Order of union does not matter

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- Order of union does not matter
- What about $\{1, 2, 3\} \cap \{3, 4, 5\}$ versus $\{3, 4, 5\} \cap \{1, 2, 3\}$?

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- $A = \{1,2,3\}$, and $B = \{3,4,5\}$. Compare $A \cap B$ and $A - B$.

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- $A \cap B = \{3\}$ and $A - B = \{1,2\}$

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- What about $\{1,2,3\} \cap \{3,4,5\}$ versus $\{3,4,5\} \cap \{1,2,3\}$?
- Both are $\{3\}$.
- $A = \{1,2,3\}$, and $B = \{3,4,5\}$. Compare $A \cap B$ and $A - B$.
- $A \cap B = \{3\}$ and $A - B = \{1,2\}$
- $A = (A \cap B) \cup (A - B)$

6.1: Summary

- Today we learned about **sets**, **union**, **intersection**, and **difference**.
- You are now ready to complete HWC1 and HWC2
- Make sure to take advantage of office hours