MA162: Finite mathematics

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University of Kentucky

March 30, 2011

Schedule:

- HW C2 is due Monday, Apr 4th, 2011.
- HW C3 is due Sunday, Apr 10th, 2011.
- Exam 3 is Monday, Apr 11th, 5:00pm-7:00pm.
- Alternate exam form (again) due Tomorrow, Mar 31, 2011.

Today we will cover 6.2: Counting

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans
- Chapter 6, Counting
  - Inclusion exclusion
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  - Multiplication principle
  - Permutations and combinations

6.2: Counting the missing piece

- Out of 100 coffee drinkers surveyed, 70 take cream, and 60 take sugar. How many take it black (with neither cream nor sugar)?
- Well, it is hard to say, right?
  30 don't use cream, 40 don't use sugar, but. . .

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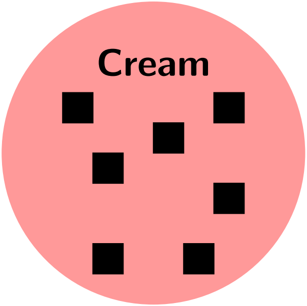

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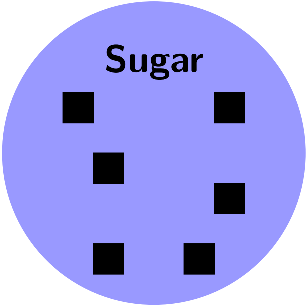

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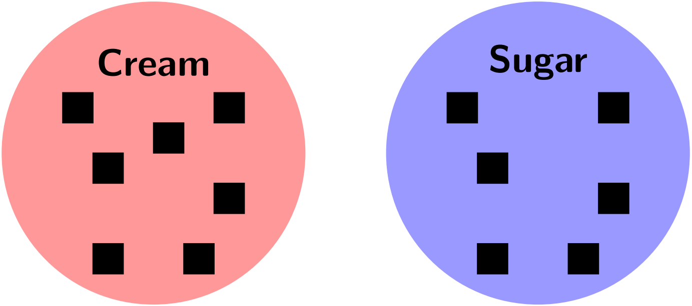

- $60 + 70 = 130$ is way too big. What happened?
  Try it yourself!

6.2: The overlap

- In order to figure out how many take it black, we need to know how many take it with cream or sugar or both.

$$\#\text{Black} = 100 - n(C \cup S)$$

- However, in order to find out how many take either, we kind of need to know how many take both:

$$n(C \cup S) = n(C) + n(S) - n(C \cap S) = 70 + 60 - n(C \cap S)$$

- So what if 50 people took both?

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- Then $n(C \cup S) = 130 - 50 = 80$ and so $100 - 80 = 20$ took neither.

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- Suppose we know that there were 200 people in the testing pool. About how many were drug users?
- Assuming exactly 5% of non-users returned positive, there is a unique answer. Let me know when you've found it.

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- All 10 are false positives; 100% wrong, but 95% accurate?
  Be careful what you are counting.

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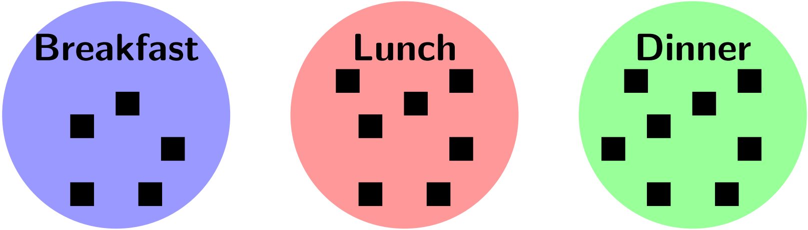
- What if those were exactly the 20 people that didn't eat dinner?
- Could be 0%, could be 50%. We need to know more!

6.2: More information and a picture

- If we let B, L, D be the sets of people, then we are given

$$n(B) = 50, n(L) = 70, n(D) = 80,$$

and we want to know $n(B \cap L \cap D)$.



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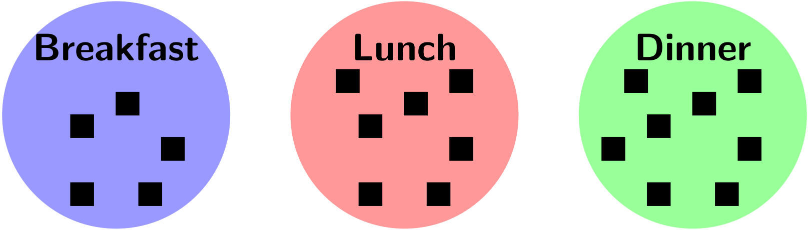

- What if we find out:

$$n(B \cap L) = 30, n(B \cap D) = 40, n(L \cap D) = 40$$

We can find the overlaps!

6.2: More information and a formula

- Just like before, there is a formula relating all of these things:

$$n(B)+n(L)+n(D)+n(B\cap L\cap D) = n(B\cup L\cup D)+n(B\cap L)+n(L\cap D)+n(D\cap B)$$

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- We plugin to get:

$$55+65+80+n(B\cap L\cap D) = 100+34+46+40$$

$$n(B\cap L\cap D) = 100+34+46+40-55-65-80 = 20$$

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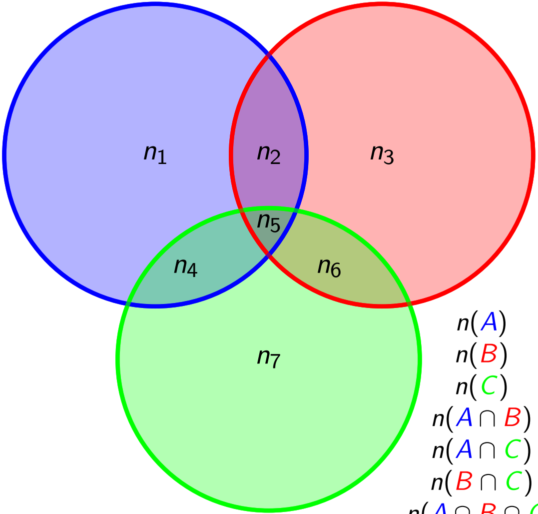
$$55 + 65 + 80 + n(B \cap L \cap D) = 100 + 34 + 46 + 40$$

$$n(B \cap L \cap D) = 100 + 34 + 46 + 40 - 55 - 65 - 80 = 20$$

- **Inclusion-exclusion formula** will be given on the exam,
  but make sure you know how to use it!

6.2: Picture and formula



$$
\begin{aligned}
n(A) &= n_1 + n_2 + n_4 + n_5 \\
n(B) &= n_2 + n_3 + n_5 + n_6 \\
n(C) &= n_4 + n_5 + n_6 + n_7 \\
n(A \cap B) &= n_2 + n_5 \\
n(A \cap C) &= n_4 + n_5 \\
n(B \cap C) &= n_5 + n_6 \\
n(A \cap B \cap C) &= n_5 \\
n(A \cup B \cup C) &= n_1 + n_2 + n_3 + n_4 \\
& \quad n_5 + n_6 + n_7
\end{aligned}
$$

6.2: Summary

- We learned the notation $n(A) =$ the number of things in the set A
- We learned the basic inclusion-exclusion formulas:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

and

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

- Make sure to complete HWC1 and HWC2