MA162: Finite mathematics

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April 18, 2011

Schedule:

- HW D1 is due Today, Apr 18th, 2011. (also taxes due)
- HW D2 is due Monday, Apr 25th, 2011.
- HW D3 is due Friday, Apr 29, 2011.
- Final Exam is Wednesday, May 4th, 6:00pm-8:00pm

Today we will cover 7.2: Probability

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- The event "rolling saved us money" is all those pairs that total to more than 6.
- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- 16 ways to win, 32 ways total, so $\frac{16}{32} = \frac{1}{2} = 50\%$ chance
- Explicitly:
  HHHHH, HHHHT, HHHTH, HHHTT, HHTTT, HTHHH,
  HTTTH, HTTTT, THHHH, THHHT, THTTT, TTHHH,
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- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?
- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- However, that's not very likely to happen and quite expensive to plan for.
- If each bulb is independent, that is $(0.1\%)^{700} \approx 0\%$ chance of this happening

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- Total is: $0.844 = 84.4\%$ chance that at most one breaks, so not too bad. Every 6 weeks you’ll have a light out and no replacement, but not too bad.

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- What are the odds that 10 is enough?
- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$,
  exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$,
  exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2(99.9\%)^{6998} \approx 2.2\%$,
  ...

0	1	2	3	4	5	6	7	8	9	10
0.1	0.6	2.2	5.2	9.1	12.7	14.9	14.9	13.0	10.1	7.0

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- Total is: $0.902 = 90.2\%$ chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

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- The larger the population, the less extreme the whims of fortune
- This is why insurance is important; the risk to one person is great
- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

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- Sample space is:
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- 12 bad out of 30 total is 40% chance for showers (of fists)