MA162: Finite mathematics

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University of Kentucky

April 25, 2011

Schedule:

- HW D2 is due Today, Apr 25th, 2011.
- HW D3 is due Friday, Apr 29, 2011.
- Final Exam is Wednesday, May 4th, 6:00pm-8:00pm.

Today we will cover 7.5: Rules of probability

Final exam breakdown

- Chapter 1 and 2: Linear systems:
  - Convert a word problem to a system of equations
  - Convert a system of equations to matrix,
    REF or RREF it,
    backsolve or read solution,
    "Free variables"
- Chapter 3 and 4: Linear optimization:
  - Convert a word problem to a system of inequalities
  - Solve a system of inequalities using the graphical method
  - Read a solution from the final tableau of a simlex algorithm
- Chapter 6 and 7: Counting and probability:
  - Inclusion-exclusion in probability
  - Fair gambling
  - Unfair?

7.5: The Punnet square of probability

- Suppose we have the following table of young men and women with and without driver's licenses:

	Yes	No	Total
M	491	9	500
F	486	14	500
T	977	23	1000

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- What are the odds that a randomly selected non-driver is female?

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- Are females less likely to be drivers?

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- Are females less likely to be drivers?
- Probability a female is a driver: $\frac{486}{500} = 97\%$ nearly the same

7.5: Conditional probability

- Let's redo this using the language of events:
  - M is the event the chosen person is male
  - F is the event the chosen person is female
  - Y is the event the chosen person has a driver's license
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- We need a name for this calculation, **conditional probability** $Pr(F|N) = Pr(N \cap F)/Pr(N)$ is the probability of F **given** N

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- We want to compare the probabilities of $Pr(A)$ versus $Pr(A|B)$ if they are equal then the events are **independent**

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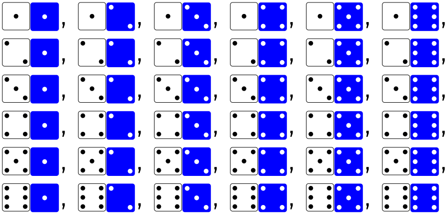

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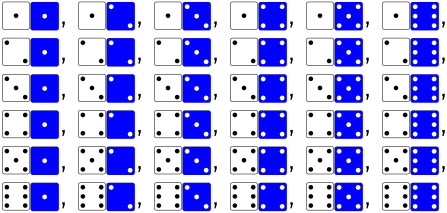

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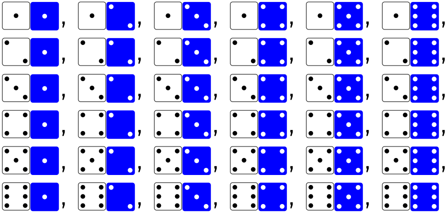

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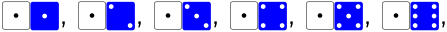

$4/6 \approx 67\%$

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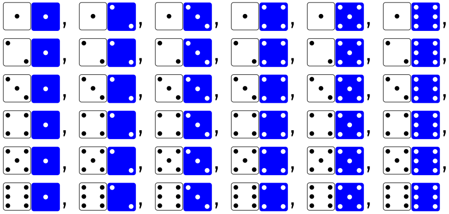

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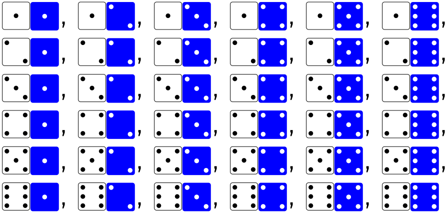

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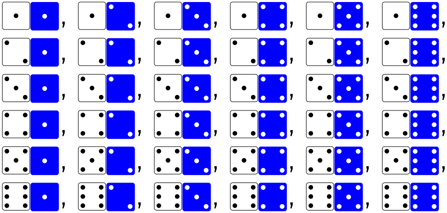

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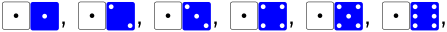

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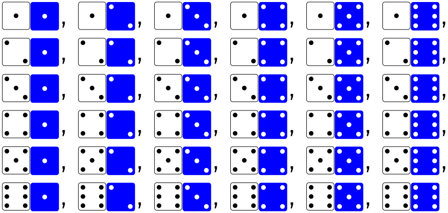

$18/36 = 50\%$

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- Just count!

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- The first die had no effect on the outcome! The two events are said to be **independent**.

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  $85/340 \approx 25\%$
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  "Mostly". The probabilities are not equal, but they are close.

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- Weighted averages

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- 45%, right?

Reasoning backwards

- Shifty Teddy is spending some time on the gameshow "Who's Gow?" and so you have to use his pal, Shifty Eddy, to run cokes for you. You end up with a coke 30% of the time. How often does he take the money and run?

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- **Bayes's Law:** $Pr(E \cap F) = Pr(F|E) \cdot Pr(E)$ – a weighted average!