MA111: Contemporary mathematics

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Schedule:

- Participation quiz on BB should be done today (and take like 30 seconds)
- HW 0B is due Today, Aug 31st, 2011.
- HW 1A is due Friday, Sep 2nd, 2011.
- Exam 1 is Monday, Sep 12th, during class.

Today we will look at how vote counters can get more out of rankings.

Review: Strategic voting

- Counting just first place votes can elect a majority loser
- The majority can do something about it; they can lie about who they think is best
- In class, we found a good strategy: count first place votes, but if they tie, count second place votes.
 See exercise #74 in book for more.
- Anytime there is a non-unanimous vote, somebody loses
- Can we encourage people to tell the truth when they vote?
- Can we take into account more than just first place votes?

| • | Explain | why A | is the | undisputed | champeen: |
|---|---------|-------|--------|------------|-----------|
|---|---------|-------|--------|------------|-----------|

| 13 | 12 | 6 | 3 |
|----|----|---|---|
| В | D | А | D |
| С | А | С | С |
| Α | С | В | В |
| D | В | D | А |

• Explain why A is the undisputed champeen:

• Is there any candidate that can beat A?



| | | 13 | 12 | 6 | 3 |
|------|---|----|----|---|---|
| | | В | D | А | D |
| • E> | plain why A is the undisputed champeen: | С | А | С | С |
| | | А | С | В | В |
| | | D | R | D | Δ |

- Is there **any** candidate that can beat A?
- Between A and D, the first and third columns prefer A, that is 19 to 15.
 Between A and C, the second and third columns prefer A, that is 18 to 16.
 Between A and B, the second and third columns prefer A, that is 18 to 16.
- A is called a **Condorcet winner** because it beats every other candidate in a head-to-head matchup. It is better than all the rest. Surely it should win?

| | 13 | 31 | .2 | 6 | 3 |
|---|-------|----|----|---|---|
| | E | 3 | D | А | D |
| • Explain why A is the undisputed champed | en: (| 2 | А | С | С |
| | A | ١ | С | В | В |
| | Г |) | R | D | Δ |

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- A is called a **Condorcet winner** because it beats every other candidate in a head-to-head matchup. It is better than all the rest. Surely it should win?
- **Condorcet fairness criteria** simply says a Condorcet winner should, you know, actually win the election

Activity: Let's keep score



- D just killed in the second and fourth columns, beating (12)(1+1+1)+(3)(1+1+1) candidates. Of course that is it.
- How many candidates did C beat? How about B and A?

Activity: Let's keep score



- D just killed in the second and fourth columns, beating (12)(1+1+1)+(3)(1+1+1) candidates. Of course that is it.
- How many candidates did C beat? How about B and A?
- Well C has the high score, so C is the Borda winner

- In track-and-field, swimming, and similar sports, we have lots of rankings
- Maybe Alex has one gold and three bronze and Bob has four silvers
- Who did "better"? Can we choose a winner?
- Do we just count gold medals, or should we count the silvers and the bronze? In Borda count, we count them all, down to last place.
- In Borda count everyone gets a trophy, so its more fair.

- Fairness criteria: how do plurality and Borda count compare?
- Do they always let the Condorcet winner win?

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• What happens in this election?

| | 10 | 6 |
|-----|----|---|
| 1st | A | В |
| 2nd | В | С |
| 3rd | C | А |

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• What happens in this election?

- A has more than 50% of the first place vote, A is a majority winner.
- A beats B 10 to 6, and A beats C 10 to 6, so A is also a Condorcet winner.

| | 10 | 6 |
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- Do they always let the Condorcet winner win?

• What happens in this election?

- A has more than 50% of the first place vote, A is a majority winner.
- A beats B 10 to 6, and A beats C 10 to 6, so A is also a Condorcet winner.
- But A only beat (10)(1+1) + (6)(0) = 20 candidates, while B beat (10)(1) + (6)(1+1) = 22 candidates. B has the high score, B is the **Borda winner**

| | 10 | 6 |
|-----|----|---|
| 1st | Α | В |
| 2nd | В | С |
| 3rd | C | А |

Winners and fairness

- Majority winner has more than 50% of the first place votes
- **Plurality winner** has the most first place votes
- Condorcet winner beats all other candidates one-on-one
- **Borda winner** beats more candidates than any other candidate (high score)

- Majority fairness criterion says a Majority winner should win
- Condorcet fairness criterion says a Condorcet winner should win

- Vocab: Majority winner, Condorcet winner, Plurality winner, Borda winner
- Participation (15%): There is a quiz on blackboard, under **Assignments**. Should do it today. Due by Thursday.
- Read section 1.3 of the textbook.
- Online homework (30%):
 - 0B is due Today.
 - 1A is due Friday. Should do it today.