MA111: Contemporary mathematics

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Schedule:

- Participation quiz on BB should be done **today** (and take like 30 seconds)
- HW 1A is due Today, Sep 2nd, 2011.
- HW 1B-1F are due next Friday, Sep 9th, 2011. (That's a lot)
- Exam 1 is Monday, Sep 12th, during class.

Today we will look at how to simplify elections and check whether it is fair.

Review: Everybody is a winner

- Four kinds of winners: Majority, Condorcet, Plurality, Borda
- Not always a majority or Condorcet winner, but if there is, it seems unfair (or at least strange) for them to lose
- Borda gives everyone points based on how they did; high score wins
- Plurality just counts first place votes

What happens if we eliminate irrelevant candidates?

- If we just asked people for their first place votes (and they were honest), then C wouldn't get any votes. Why not get rid of C?

13	12	6	3
B	D	A	D
C	A	C	C
A	C	B	B
D	B	D	A

$\longrightarrow$

13	12	6	3
B	D	A	D
A	A	B	B
D	B	D	A

- If the polls had only asked first place candidates, then C might not have even realized they had a chance!

Why not keep eliminating?

- A only got 6 votes, so is not a real contender, right? DELETED.

13	12	6	3
B	D	A	D
A	A	B	B
D	B	D	A

$\longrightarrow$

13	12	6	3
B	D	B	D
D	B	D	B

$\xrightarrow{=}$

19	15
B	D
D	B

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- In presidential primaries, candidates often drop out of the race as soon as they (or their financial backers) think they are going to lose

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$\xrightarrow{=}$

19	15
B	D
D	B

- With A gone, the 6 voters swing the election to B, and B wins
- In presidential primaries, candidates often drop out of the race as soon as they (or their financial backers) think they are going to lose
- If the voters themselves were steady in the opinions, then this would result in **plurality with elimination**

Unfortunately voters change their minds too

- Secret poll before the elimination election. Who will win?

10	8	7	4
A	B	C	C
C	A	B	A
B	C	A	B

Unfortunately voters change their minds too

- Secret poll before the elimination election. Who will win? A

10	8	7	4
A	B	C	C
C	A	B	A
B	C	A	B

$\xrightarrow{\text{B lost}}$

10	8	7	4
A	A	C	C
C	C	A	A

$\xrightarrow{\text{combine}}$

18	11
A	C
C	A

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A	A	C	C
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$\xrightarrow{\text{combine}}$

18	11
A	C
C	A

- 4 voters on the end overhear the results, and change their mind:

10	8	7	4
A	B	C	C
C	A	B	A
B	C	A	B

$\xrightarrow{\text{lie}}$

10	8	7	4
A	B	C	A
C	A	B	C
B	C	A	B

- Now who wins?

Unfortunately voters change their minds too

- Secret poll before the elimination election. Who will win? A

10	8	7	4
A	B	C	C
C	A	B	A
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$\xrightarrow{\text{B lost}}$

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$\xrightarrow{\text{C lost}}$

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$\xrightarrow{\text{C lost}}$

10	8	7	4
A	B	B	A
B	A	A	B

- Now who wins? B wins!
- Voters tried to help A, but made A lose. This violates the **monotonicity criterion**.

Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll?

7	6	5
A	B	C
B	C	A
C	A	B

Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

7	6	5
A	B	C
B	C	A
C	A	B

$\xrightarrow{\text{C lost}}$

7	6	5
A	B	A
B	A	B

$\rightarrow$ A wins 12 to 6

Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

7	6	5
A	B	C
B	C	A
C	A	B

$\xrightarrow{\text{C lost}}$

7	6	5
A	B	A
B	A	B

$\rightarrow$ A wins 12 to 6

- 2 of B's supporters just give up and don't vote. Who wins?

7	4	5
A	B	C
B	C	A
C	A	B

Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

7	6	5
A	B	C
B	C	A
C	A	B

$\xrightarrow{\text{C lost}}$

7	6	5
A	B	A
B	A	B

$\rightarrow$ A wins 12 to 6

- 2 of B's supporters just give up and don't vote. Who wins? C

7	4	5
A	B	C
B	C	A
C	A	B

$\xrightarrow{\text{B lost}}$

7	4	5
A	C	C
B	A	A

$\rightarrow$ C wins 9 to 7

Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

7	6	5
A	B	C
B	C	A
C	A	B

$\xrightarrow{\text{C lost}}$

7	6	5
A	B	A
B	A	B

$\rightarrow$ A wins 12 to 6

- 2 of B's supporters just give up and don't vote. Who wins? C

7	4	5
A	B	C
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$\xrightarrow{\text{B lost}}$

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B	A	A

$\rightarrow$ C wins 9 to 7

- By not voting at all, they got a better result (their 2nd place pick)

A goofy Borda count

- For some reason A through Z all got nominated as candidates.
- 9 voters love A, and like B, and C-Z are like so whatever.
- 1 voter is obsessed with B, and decides to lie on his vote and give A his last place vote. What happens?

9	1
A	B
B	C
C	D
⋮	⋮
Y	Z
Z	A

The results of one crazy voter

9	1
A	B
B	C
C	D
⋮	⋮
Y	Z
Z	A

- A gets $(9)(25) = 225$ points, B gets $(9)(24) + 25 = 241$ points
- One (crazy) voter managed to change the entire election!
- This is only possible when the number of candidates is large compared to the number of voters (a pretty silly situation, but one faced by some small clubs)

One crazy election

- We've seen a lot of different "winners" here:

13	12	6	3
B	D	A	D
C	A	C	C
A	C	B	B
D	B	D	A

- Majority: none
- Condorcet: A
- Plurality with elimination: B
- Borda: C
- Plurality: D

Homework

- Vocab: Majority winner, Condorcet winner, Plurality winner, Borda winner
- Participation (15%): There is a quiz on blackboard, under **Assignments**. Should do it today. Due by Thursday.
- Read section 1.3 of the textbook.
- Online homework (30%):
  - 0B is due Today.
  - 1A is due Friday. Should do it today.