

# MA111: Contemporary mathematics

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September 19, 2011

## SCHEDULE:

- Participation quiz on BB should be done **today** (and take like 30 seconds)
- HW 10.2 is due Today, Sep 19th, 2011.
- HW 10.3 is due Wednesday, Sep 21st, 2011.
- HW 10.6 is due Friday, Sep 21st, 2011.
- Exam 1 is Monday, Oct 3rd, during class.

Today we will look at borrowing money for a year, 10.3, compound interest.

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- \$1000 Treasury Bill earning 10% simple interest annually  
Maturity in 6 years, present value is? \$625.00

$$F = \$1000$$

$$r = 0.010 \text{ per year}$$

$$t = 6 \text{ years}$$

$$F = P + I = P + Prt = P(1 + rt)$$

$$P = F/(1 + rt) = \$1000/(1 + 6(0.01)) = \$1000/1.06 = \$625.00$$

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- A savings bond doubles its value in 20 years.  
What is the simple interest APR? 5%

$$P = \$1$$

$$F = \$2$$

$$t = 20 \text{ years}$$

$$I = F - P = \$1$$

$$r = \frac{I}{Pt} = (\$1/\$1)/20 = 0.05 = 5\%$$

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- In November, Stanley Beard stops by, asks about the weather  
And, oh yes, his brother wants his money back

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- In December, Black Beard stops by and demands the money for his brother

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And, oh yes, his brother wants his money back
- You don't have the money,  
but Stanley Beard loans you the money at the same rate
- In December, Black Beard stops by and demands the money for his brother
- You don't fancy a wooden leg, so you offer to pay the \$130,  
\$100 plus 3 months simple interest



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- You borrowed \$110 from Red Beard, so owe him \$121
- You borrowed \$121 from Stanley Beard, so owe him \$133.10

## 10.3: Compound interest formulas

- The following formula is important enough to memorize:

P = Present value

F = Future value

p = periodic compound interest rate

T = number of periods

$$F = P(1 + p)^T$$

- Same as repeatedly doing simple interest for 1 period

## 10.3: More compound interest formulas

- These formulas are not worth memorizing, in my opinion

$P$  = Present value

$F$  = Future value

$APR = r$  = annual, nominal, compound interest Rate

$n$  = Number of periods per year

$t$  = number of years

$APY = r_{eff}$  = annual effective Yield (what you actually get)

$$F = P \left(1 + \frac{r}{n}\right)^{(nt)}$$

$$APY = \left(1 + \frac{r}{n}\right)^{(n)} - 1$$

- If  $n = \infty$ , then we get:

$$F = Pe^{(rt)}$$

## 10.3: Viewing the pirates as a single bank

- If we view all three brothers as the same lending institution

$$P = \$100$$

$$p = 10\% \text{ per month}$$

$$T = 3 \text{ months}$$

$$\begin{aligned} F &= P(1 + p)^T = \$100(1 + 0.10)^3 = \$100(1.1)^3 \\ &= \$100(1.331) = \$133.10 \end{aligned}$$

$$I = \$33.10$$

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- A few investments compound annually
- If you have \$100 in a savings account earning 1% APR, compounded annually,  
How much is it worth in 4 and a half years?



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- If you have \$100 in a savings account earning 1% APR, compounded annually,  
How much is it worth in 4 and a half years? \$104.06, not \$104.58

$$P = \$100$$

$$p = 0.01$$

$$T = 4 \text{ not } 4.5$$

$$F = P(1 + p)^T = \$100(1.01)^4 = \$104.060401 = \$104.06$$

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- Pirates charge 120% APR, because 12 months in the year
- APR is simple, but not all that useful
- APY simply asks “What happens to your dollar after a year?”

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- Pirates charge 120% APR, because 12 months in the year
- APR is simple, but not all that useful
- APY simply asks “What happens to your dollar after a year?”
- 1% APR compounded annually is 1% APY
- 1% APR compounded monthly is 1.005% APY
- 1% APR compounded every second is 1.005% APY

## 10.3: APY for consumer loans

- So APR/APY doesn't matter much now for the banks
- But for consumers interest rates are still high:
- 5% APR compounded continuously is 5.127% APY  
Mortgage
- 25% APR compounded continuously is 28.4% APY  
Credit card
- 150% APR compounded bi-monthly is 281.47% APY  
Alternating between two pawn shops

# Homework

- Calculations: compound interest, APY
- Participation (15%): There is a quiz on blackboard, under **Assignments**. Should do it today. Due by Sunday.
- Read section 10.3 of the textbook. Skim 10.4 - 10.6.
- Online homework (30%):
  - HW 10.2 is due Today.
  - HW 10.3 is due Wednesday. Really long.
  - HW 10.6 is due Friday.