#### MA111: Contemporary mathematics

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September 21, 2011

Schedule:

- Participation quiz on BB should be done today (and take like 30 seconds)
- HW 10.3 is due Today, Sep 21st, 2011.
- HW 10.6 is due Friday, Sep 21st, 2011.
- Exam 1 is Monday, Oct 3rd, during class.

Today we will look at borrowing money for several years, 10.6, amortized loans.

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F =\$100(1.024)(1.024)(1.021)(1.021)(1.021) = \$111.60

 \$100 Savings Account earning 2.4% compound interest annually the first two years, then you deposit another \$100, then 2.4% compound interest annually the next three years, Value after 5 years?

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Т	F formula = $F$	number
Now	\$100.00 =	\$100.00
1st year	(\$100.00)(1.024) =	\$102.40
2nd year	(\$102.40)(1.024) =	\$104.86
deposit	104.86 + 100 =	\$204.86
3rd year	(\$204.86)(1.024) =	\$209.78
4th year	(\$209.78)(1.024) =	\$214.81
5th year	(\$214.81)(1.024) =	\$219.97

 \$100 Savings Account earning 2.4% compound interest annually the first two years, then you deposit another \$100, then 2.4% compound interest annually the next three years, Value after 5 years? \$219.97 or \$219.96

• Faster is to think: \$100 was compounded 5 years, plus \$100 was compounded 3 years

F =\$100(1.024)<sup>5</sup> + \$100(1.024)<sup>3</sup> = \$219.9641731 = \$219.96

## Installment loans

- What if Black Beard only needed \$20? Maybe he'd loan you the rest...
- You owed \$133.10 to Stanley, so after paying \$20 to Mr. Beard, you owe \$113.10, which Black Beard could loan you (to pay back Stanley)
- Next month, maybe Red comes back and only needs \$20, so you owe Black Beard (\$113.10)(1.1) = \$124.41 and pay him back \$20, so that is \$104.41 that Red is loaning you.
- If this continued month after month, the amount you owed would go down slowly (not \$20 a month, only \$8.69 the Black-Red month)
- How long does it take to finally pay it off?

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- How long does it take to finally pay it off?
   I get 7 to 8 months, eighth month only costs \$13.72 (not \$20)

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• 
$$P = F/(1+p)^T = $331/(1.1)^3 = $248.69$$

#### 10.6: Longer installment loans

- What if it was 20 payments? Add them by hand?
- We just use the formula:

$$P = Mq rac{1-q^T}{1-q} \quad ext{where } q = rac{1}{1+p}$$

- Here *M* is the monthly (periodic) payment, and *p* is the periodic interest rate
- Be careful not to round q (keep 6 to 10 digits)
- For the pirates,  $q = \frac{1}{1+0.1} = 1/1.1 = 0.90909090$

$$P = 100(0.90909090) \frac{1 - 0.90909090^3}{1 - 0.909090909} = \$248.69$$

• What if we needed to borrow \$300 instead. What would the payment be?

$$300 = M(0.90909090) \frac{1 - 0.90909090^3}{1 - 0.90909090} = 2.486851942M$$

so

$$M =$$
\$300/2.48685 = \$120.63

not much more.

#### Homework

- Calculations: installment loans (what happens), installment loans (calculating the payment)
- Participation (15%): There is a quiz on blackboard, under **Assignments**. Should do it today. Due by Thursday.
- Read section 10.6 of the textbook. Skim 10.4 10.5.
- Online homework (30%):
  - HW 10.3 is due Today. Really long.
  - HW 10.6 EZ is due Friday. 2 problems, more or less use 10.3 stuff to solve them.