#### MA111: Contemporary mathematics

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Schedule:

- Participation quiz on BB should be done today (and take like 30 seconds)
- HW 10.6 is due Wednesday, Sep 28th, 2011.
- Exam 2 is Monday, Oct 3rd, during class.

Today we will look at saving money, briefly, and then work lots of problems on installment loans.

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• After 3 months, you have

 $100(1.1)^2 + 100(1.1) + 100$ 

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• The right hand side is a pain for T = 4 but much easier for T = 12

$$100\frac{1.1^4 - 1}{0.1} = 100\frac{0.4641}{0.1} = 464.10$$

$$100\frac{1.1^{12}-1}{0.1} = 100\frac{2.13843}{0.1} = 2138.43$$

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- First is a \$2000.00 monthly payment, second is the same money upfront, but \$1974.58 monthly payment

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(\$460.24)(1200) - \$100000 = \$452288.00