MA111: Contemporary mathematics

Jack Schmidt

University of Kentucky

October 7, 2011

Schedule:

- Exam 3 is Monday, Oct 24th, during class.

Today we will introduce some graphs and Euler circuits.

5.2: First definitions

- A **graph** is made up of two pieces:
  its **vertex set** and its **edge set**.
- The **vertex set** only answers the question
  "Is X a vertex of this graph?"
  The answer is simply yes or no.
- The **edge set** only answers the question
  "How many edges connect X and Y in this graph?"
  The answer is a non-negative integer.
- There are lots of ways of writing down enough to answer these questions

5.2: List way

- We decide to only allow vertices that can be symbolized by letters

  We list the letters that are vertices
  "The vertex set is { A, B, C, F }."

- The edge set can be handled similarly.
  If A and B are connected by one edge we write down "AB".
  "The edge set is { AB, AC, BF, CF }."

5.2: List way

- We decide to only allow vertices that can be symbolized by letters

  (or something similarly write-down-able)

  We list the letters that are vertices and don't mention the ones that are not

  "The vertex set is { A, B, C, F }."
  One minor problem: the order does not matter
  "The vertex set is { C, A, B, F }."

  Could require alphabetical order

- The edge set can be handled similarly.
  If A and B are connected by one edge we write down "AB".

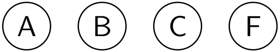
  "The edge set is { AB, AC, BF, CF }."

  Three minor problems:

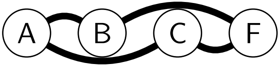
  - What if A and B are connected by three edges?
    One way is to list AB three times.
    "The edge set is { AB, AB, AB, AC, BF, CF }."
  - We can write down the same edge two different ways.
    "AB" and "BA" both mean A and B are connected by an edge
    Could require alphabetical order
  - The order of the edges doesn't matter:
    "The edge set is { CF, AB, BF, AC }."
    Could require alphabetical order

5.2: Picture way

- Each vertex is a big dot



- Each edge is a (possibly curved) line between the two dots



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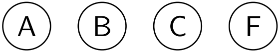

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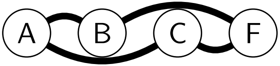
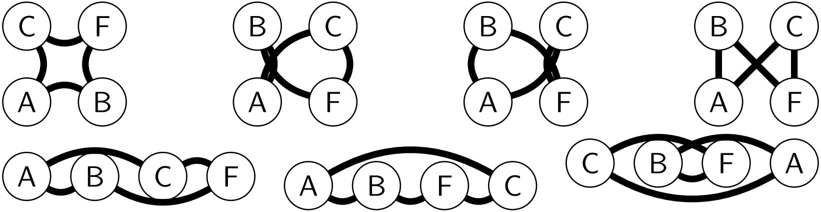


- Problems are that the same graph can be drawn very differently:



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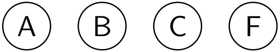

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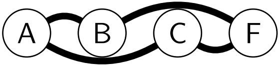

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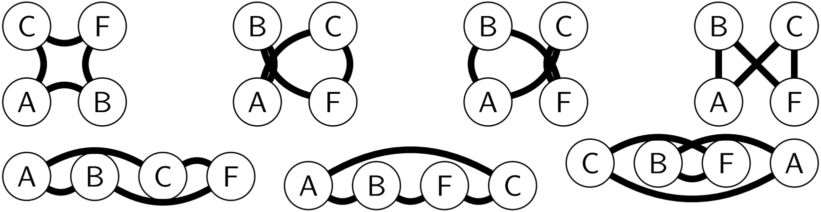
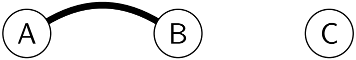


- (Alphabetical) List way is best to decide equality
  (Good, pretty) Picture way is best to reason quickly

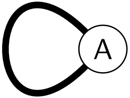
5.2: Funny vertices, funny edges

- **Isolated vertex**: A vertex that is not connected to any vertices

  Vertex set: $\{A, B, C\}$.
  Edge set: $\{AB\}$



- **Loop**: An edge that connects a vertex to itself.



  Vertex set: $\{A\}$.
  Edge set: $\{AA\}$.

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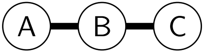
- Draw the graph you've been given on the back of it
  (In the "Picture" format)
- Figure out whose graph you've got

5.3: Definitions

- **Adjacency**: Two vertices are adjacent if the edge set says at least one edge connects them. Two edges are adjacent if there is a vertex connected by both of them.
  Vertex set: $\{A, B, C\}$.
  Edge set: $\{AB, BC\}$.



- A and B are adjacent, B and C are adjacent,
  A and C are NOT adjacent

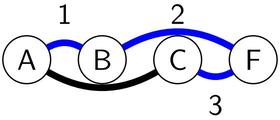
- AB and BC are adjacent

5.3: More definitions

- **Degree**: Basically counts how many edges connect to a specific vertex, but loops count twice.

- **Path and circuit**: A sequence (ordered list) of edges, so that each edge is adjacent to the next one. If the “start” and “end” of the sequence is the same vertex, then we call it a circuit, otherwise a path. We require that no edge is used more often than it occurs in the edge set.

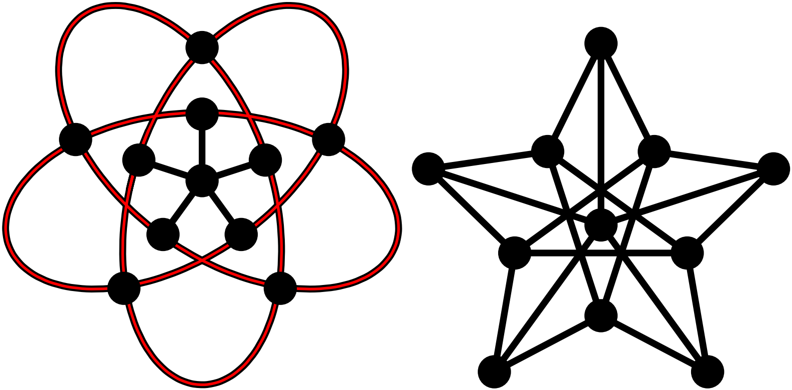
  Vertex set: $\{A, B, C, D\}$
  Edge set: $\{AB, AC, BF, CF\}$
  Path: AB-BF-CF



5.3: Even more definitions

- **Connected**: A graph is connected if every two distinct vertices are connected by a path
- **Euler path and circuit**: An Euler path or circuit is a path or circuit that uses **all** the edges in the graph.
- Here are some more examples of graphs.

5.3: More examples



- Draw these on your paper, but label the vertices so that the two edge sets are the same!