MA111: Contemporary mathematics

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Schedule:

- HW Ch 5 Part One is due Wed, Oct 12th. (little long, but easy)
- HW Ch 5 Part Two is due Wed, Oct 19th.
- Exam 3 is Monday, Oct 24th, during class.
- Exams not graded yet (and this week is busy)

Today we will go over graph models.

5.2: First definitions

- A **graph** is made up of two pieces:
  its **vertex set** and its **edge set**.

- The **vertex set** only answers the question
  "Is X a vertex of this graph?"
  The answer is simply yes or no.

- The **edge set** only answers the question
  "How many edges connect X and Y in this graph?"
  The answer is a non-negative integer.

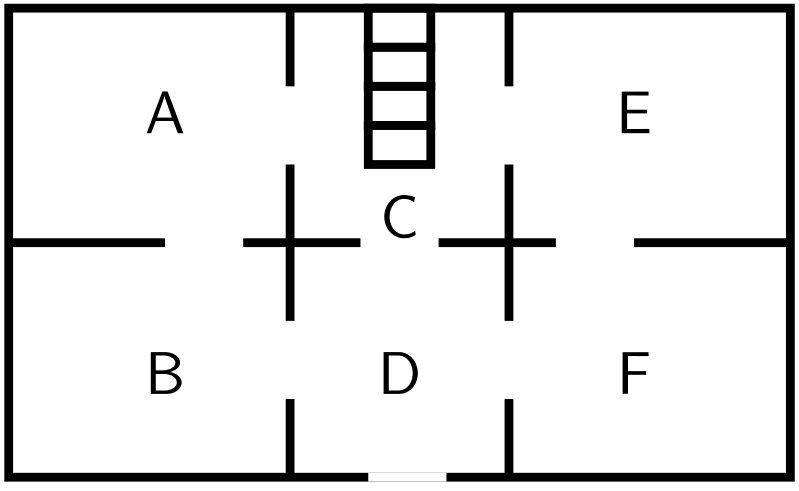
- Our goal this chapter is to cover the edge set by a path.

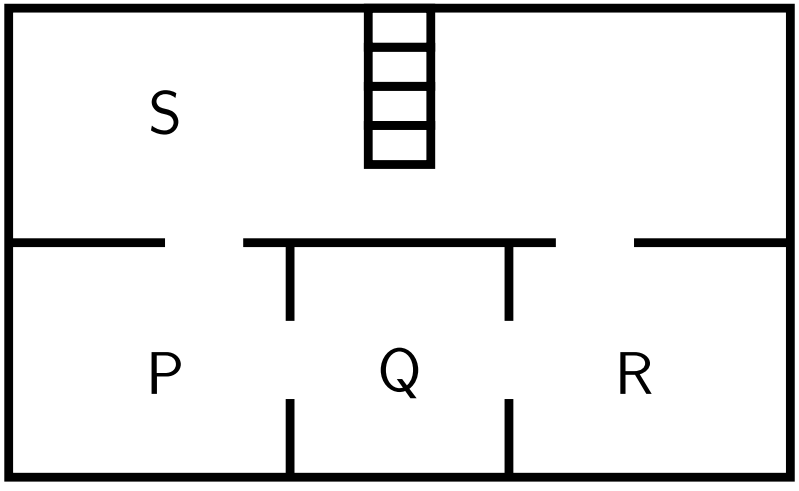
5.3: One more definition

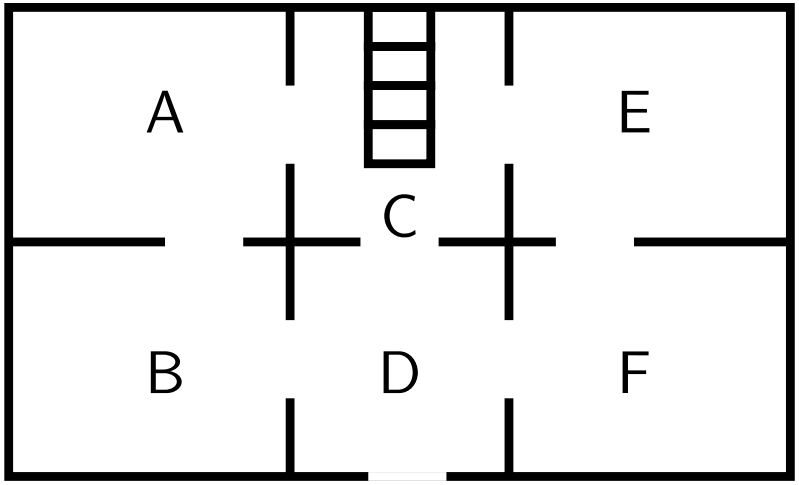
- **bridge**: An edge in a graph is a "bridge" if removing it disconnects the graph. You might want to think of it as "the last bridge".
- Imagine a two-story house with one staircase
- The Feng Shui dragon's graph would have the (doorway to the) staircase as a bridge
- If the stairs are blocked, the dragon cannot change floors
- The two floors would not be connected
- There would be no Euler path!

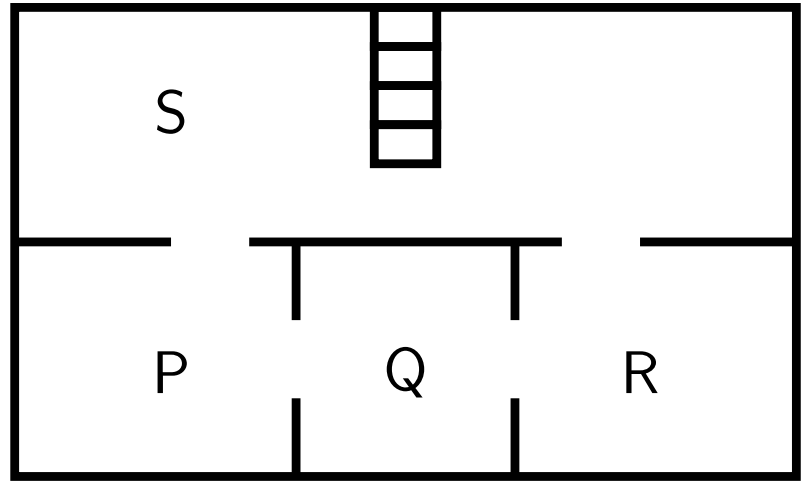
5.3: Bridge

- Dragons love stairs
- Here is a house with stairs:

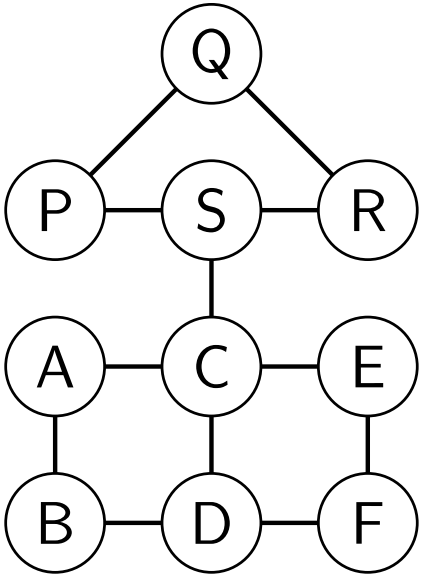


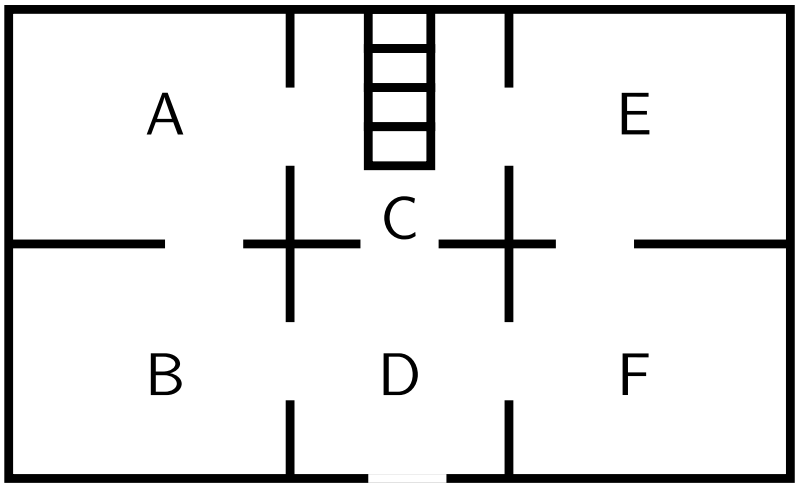


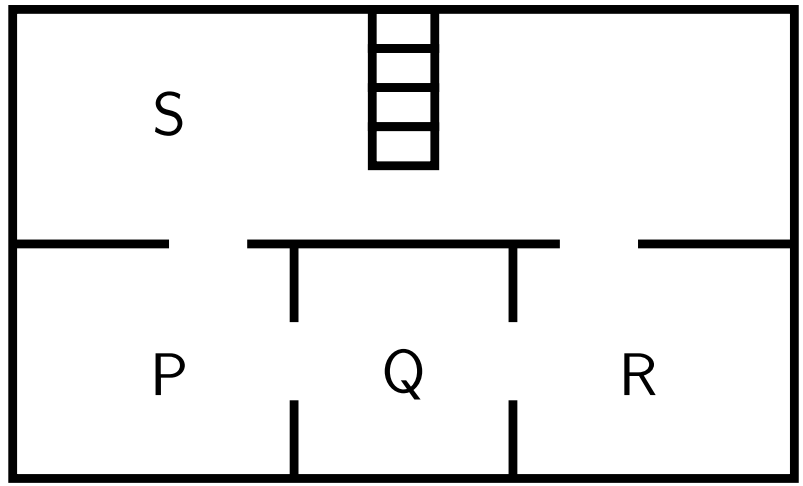




Here is the graph model:







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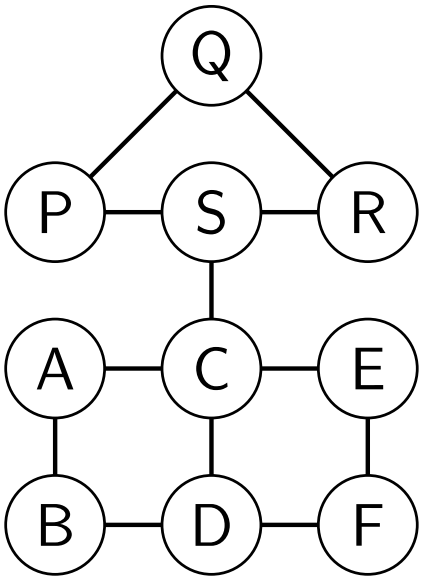
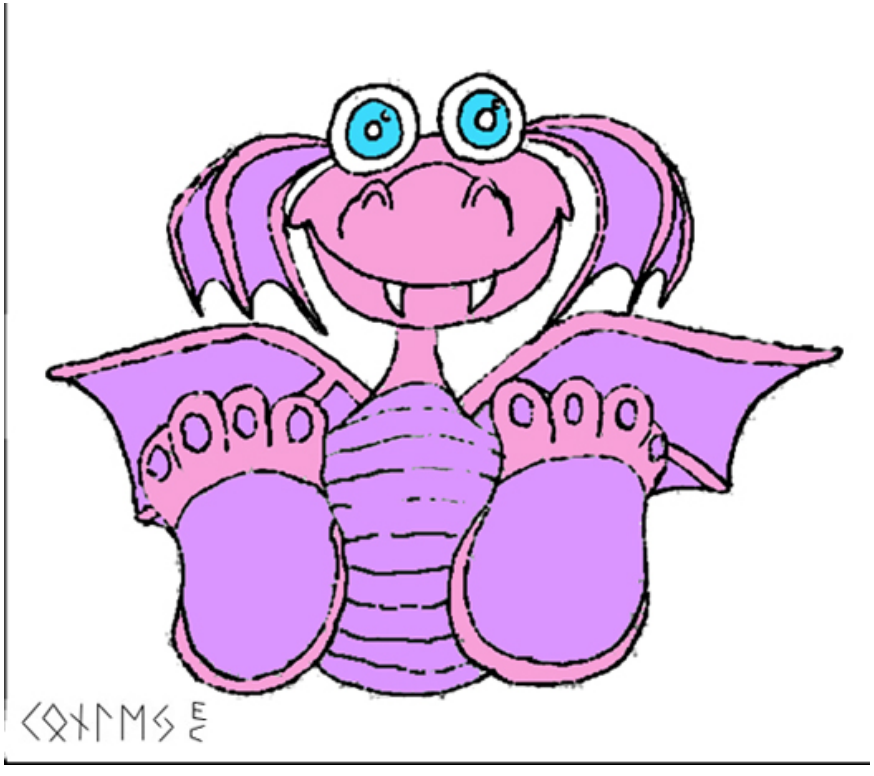
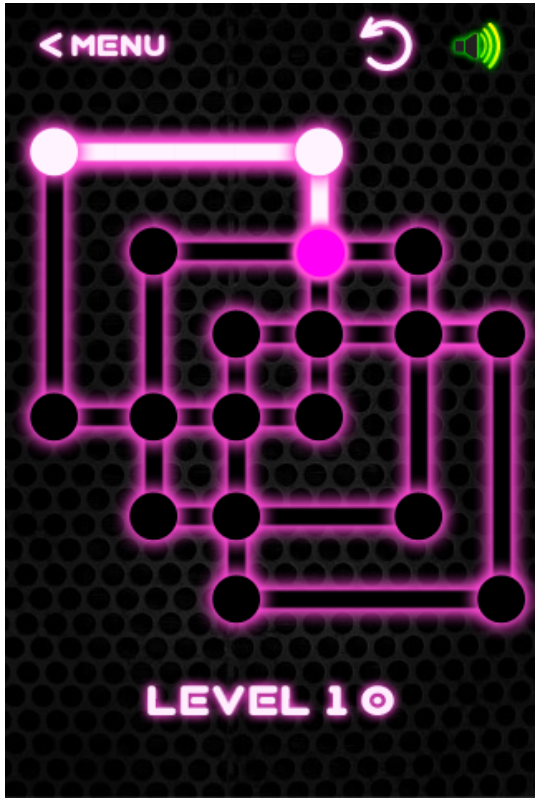


That's why the dragon is still on the stairs!

5.4: Why do we care?

- To me the most important reason is it is fun
- Be the dragon, trace the path



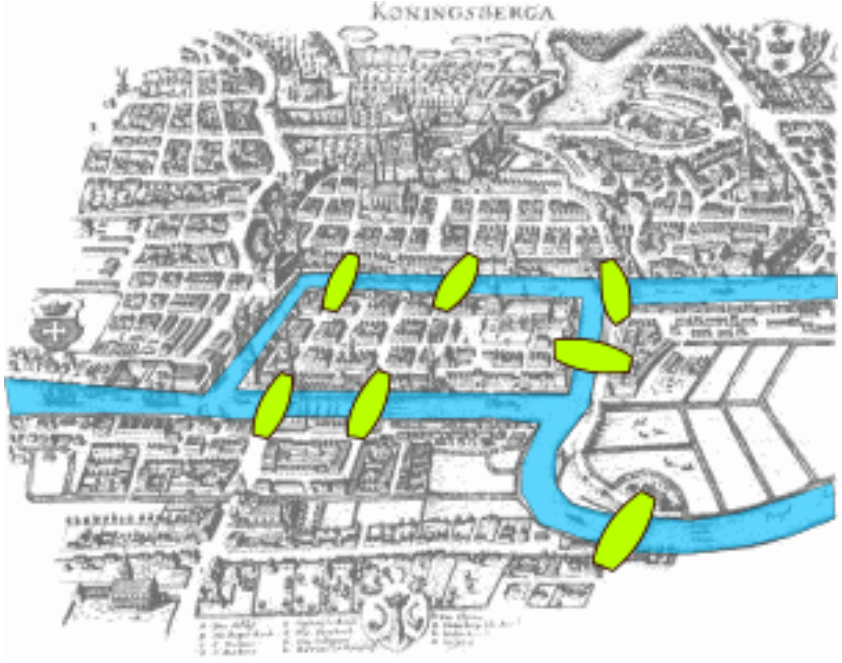


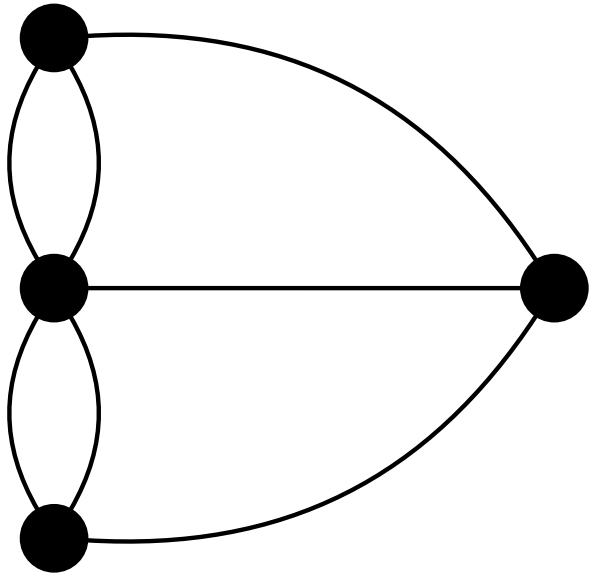
5.4: Well why do dull people care?

- There are a few practical applications:
- **Maze running:** if you need to map a maze using only a few stones (per intersection), then this chapter is for you!
- **Patrolling:** if you walk a beat, and you don't like retracing your steps all that much, then this chapter has some techniques for you.
- **Atomic bonds:** a few modern configurations of atoms reach minimum energy (stop blowing up) by finding euler paths on a spherical triangulation. If you plan to return to life as a carbon atom in a giant bucky ball, you better memorize this chapter.
- **Genomic mapping:** Euler paths handle DNA sequence matching where sequences repeat, but the two donors are not clones. If you plan on sequencing a new genome you should study this chapter. And a lot of biology.

5.4: The original problem

- In 1736, Euler published a study on walking in the park
- He wandered around Königsberg and liked the bridges.
- No one knew how to cross all 7 bridges exactly once.





- Euler gave a simple solution, and developed a new field of geometry

5.4: Textbook problems

- The textbook also mentions a security guard and a mail carrier
- They wander the streets of a town, trying to cover them evenly
- They have slightly different goals, so the graph models are slightly different
- See the textbook p. 177 (or the board) for examples

5.4: Shall we play a game?

- Design three graphs and prepare to challenge your neighbor
- One should be obviously impossible to trace
  (no Euler path or circuit)
- The other two should look possible, but one should be impossible and one should be possible
- After you have created, trade with someone nearby, and see if they can figure out which is which
- We'll bring coolest and trickiest graphs to the board

5.4: An anonymous contribution