MA111: Contemporary mathematics

Jack Schmidt

University of Kentucky

October 14, 2011

Schedule:

- HW Ch 5 Part Two is due Wed, Oct 19th.
- HW Ch 5 Part Three is due Mon, Oct 24th.
- Exam 3 is Monday, Oct 24th, during class.
- Exams not graded yet (this week is busy; but will be done for midterms)

Today we will actually find the Euler paths and circuits

5.5: The theorems

Theorem (Euler, 1736)

The sum of the degrees in a graph is twice the number of edges, so is even.

Theorem (Euler, 1736 and Hierholzer, 1873)

A graph has an Euler circuit if and only if it is connected and every vertex has even degree.

Corollary

A graph has an Euler path if and only if it is connected and exactly two of its vertices have odd degree.

5.5: Test it

- Degrees are 1,1,1

5.5: Test it

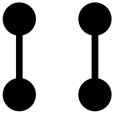
- Degrees are 1,1,1 **Impossible. No such graph.**

5.5: Test it

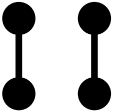
- Degrees are 1,1,1 **Impossible. No such graph.**

- Degrees are 1,1,1,1

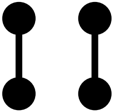
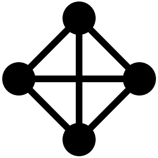
5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.

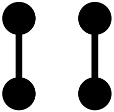
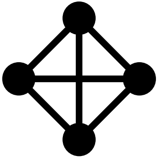
5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.
- Degrees are 3,3,3,3

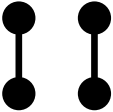
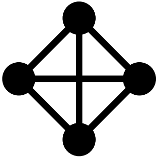
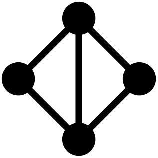
5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.
- Degrees are 3,3,3,3  No Euler path. No Euler circuit.

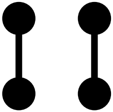
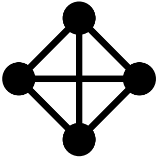
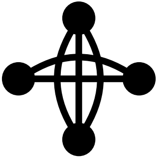
5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.
- Degrees are 3,3,3,3  No Euler path. No Euler circuit.
- Degrees are 3,3,2,2

5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.
- Degrees are 3,3,3,3  No Euler path. No Euler circuit.
- Degrees are 3,3,2,2  Has Euler path if connected.

5.5: Test it

- Degrees are 1,1,1 **Impossible. No such graph.**
- Degrees are 1,1,1,1  No Euler path. No Euler circuit.
- Degrees are 3,3,3,3  No Euler path. No Euler circuit.
- Degrees are 3,3,2,2  Has Euler path if connected.
  If it is not connected, then it has multiple edges or loops.

5.6: What more could we want?

- Euler (and Hierholzer) gave us a super easy test:
  check connectedness and then count the odd vertices:
  0 odds = Euler Circuit;
  2 odds = Euler Path;
  4,6,8 odds = Neither kind;
  1,3,5 odds = No such graph
- What more could we want?
- A similar example: Every Wed and Sat the "powerball lottery" chooses 6 numbers
- Easy to check whether there is a winning 6-number thing: is it Wed or Sat?
- You could win 20 millllllion dollars with this knowledge
- Except for one small problem

5.6: What constitutes an answer

- "Does this graph have an Euler circuit?"
- "Yes" is a complete answer, but it'd be nice to actually see one.
- "Is there a way to win the lottery tonight?"
- "Yes" is a complete answer, but it'd be nice to actually know the way
- 5.5 just answered yes or no (and why), but not HOW
- 5.6 gives one answer of how (we talked about another way Friday, exercise #67-68)

5.6: Fleury's apparently simple "how"

- Take two copies of the graph:
  Left one: erase edges as you use them Right one: label edges as you use them
- Step one: Start at an odd vertex (or anywhere if none)
- Repeat step: Erase an edge connected to the current vertex that is not a bridge; write down that edge on the right
- Termination: When the left graph is erased, you have an Euler circuit/path. If part of the graph is left, then the graph was not connected.
- Now let's work some examples

5.6: The only hard part of Fleury

- In the repeat step, you aren't allowed to remove bridges
- What if every edge is a bridge?
  (Can't happen, but why not?)
- How do you tell if something is a bridge?
  Need to make sure your left graph is "pretty" so you can just look
- Checking for bridges is very hard for a computer
  Impossible inside the corn maze
- We might prefer other algorithms

Trémaux's algorithm

- The goal is to walk every path of the maze exactly twice (once in each direction)
- At each junction, mark the way you came from.
- If that way is only marked once (as "incoming"), then turn around, mark it a second time ("outgoing") and take it to a new junction.
- If the way is marked twice, then take any way marked 1 or fewer times, mark it (either for the first time, or for the second time), and take it to a new junction.
- If you reach the end of the maze, then the single marked paths are direct route back to the start
- "Robot mice" use this to solve a maze: first time is slow; then fast