MA111: Contemporary mathematics

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Schedule:

- HW Ch 5 Part Two is due Today, Oct 19th.
- HW Ch 5 Part Three is due Mon, Oct 24th.
- Exam 3 is Monday, Oct 24th, during class.
- Exams not graded yet (this week is busy; but will be done for midterms)
- Look for practice exam tonight; work it for Friday

Today we will fix graphs without Euler paths or circuits

5.5: The theorems

Theorem (Euler, 1736)

The sum of the degrees in a graph is twice the number of edges, so is even.

Theorem (Euler, 1736 and Hierholzer, 1873)

A graph has an Euler circuit if and only if it is connected and every vertex has even degree.

Corollary

A graph has an Euler path if and only if it is connected and exactly two of its vertices have odd degree.

But what if we **really** want an Euler circuit??

5.7: Just do it

- **Exhaustive route** - a circuit that uses all the edges at least once
- **Optimal exhaustive route** - an exhaustive route of shortest possible length
- Security guard and postman have to cover all the roads
  "But boss, there are vertices of odd degree!"
  "I'll give you a vertex of odd degree if you don't get out there!"
- If we can't get a perfect answer (Euler circuit),
  then we get a "best" answer (Optimal Exhaustive Route)
- **Eulerizing:** Double edges on the graph until no odd degrees

5.7: How to do it?

- The book wimps out: "just try it"
- The real answer was a revolutionary invention from 1965:
  1. Apply the Floyd-Warshall algorithm to find shortest paths
  2. Pair up bad vertices so that the total distance is minimized Edmonds, 1965
- Our problems will be small enough that "just try it" should work
- However, there is a nice algorithm to handle grids: just walk along the edge of the grid, and double if you need to