Practice Exam

Name: ____________________________
MA111-009
2011-10-21

Part I: Vocabulary

Match the word with its definition:

__________	Graph	__________	Euler Path
__________	Vertex Set	__________	Euler Circuit
__________	Edge Set	__________	Exhaustive Route
__________	Degree	__________	Eulerization
__________	Path	__________	Optimal Exhaustive Route
__________	Circuit	__________	Optimal Eulerization
__________	Connected	__________	Handshaking Lemma

(A) A collection of relationships with two parts: a vertex set and an edge set

(B) The list of vertices of a graph, or at least a way to tell exactly what the vertices of a graph are

(C) The list of edges of a graph, or at least a way to tell how many relationships are between any two vertices

(D) The number of times a vertex appears in the edge set; the number of edges adjacent to a vertex (where loops count twice)

(E) A sequence of edges, each adjacent to the next, that start and stop at different vertices

(F) A sequence of edges, each adjacent to the next, that start and stop at the same vertex

(G) A graph such that between any two distinct vertices there is a path

(H) A path using all the edges in a graph

(I) A circuit using all the edges in a graph exactly once

(J) A circuit using all the edges in a graph at least once

(K) A list of the edges to be used more than once in an exhaustive route

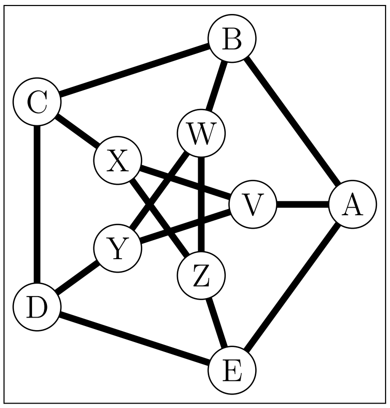
(L) An exhaustive route of shortest possible length

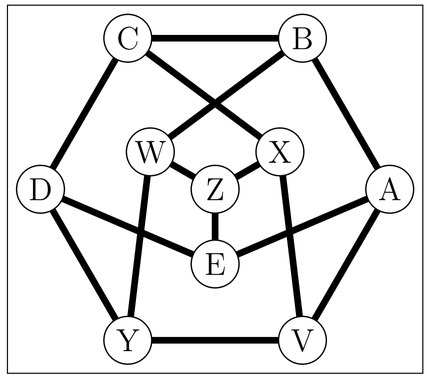
(M) An Eulerization of an optimal exhaustive route

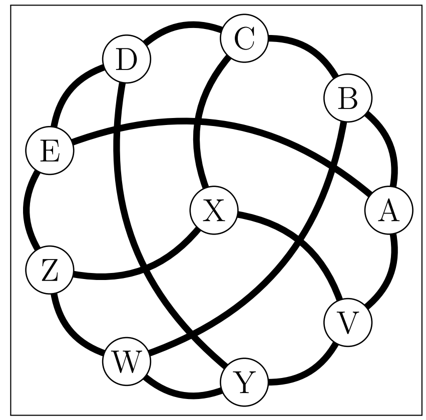
(N) The total degree is twice the number of edges

Part II: Definitions and Euler's theorem

Here are three graphs. Circle one and answer the following questions:







(a) List the vertices (alphabetically):

(b) List the edges (alphabetically):

(c) What are the degrees of the vertices?

(d) Does this graph have an Euler circuit, an Euler path, both, or neither? Why?

Part III: Handshaking lemma and graph reconstruction

1. Construct a graph with vertices of degree 3, 3, 3, 3 or explain why no such graph exists.

2. Construct a graph with vertices of degree 2, 2, 2, 2 or explain why no such graph exists.

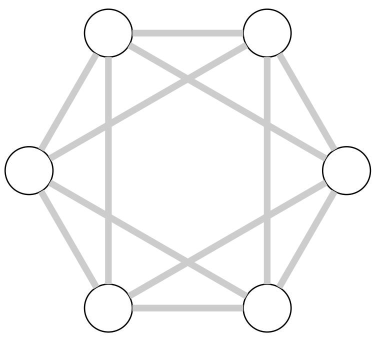
3. Construct a graph with vertices of degree 3, 3, 3, 3, 3 or explain why no such graph exists.

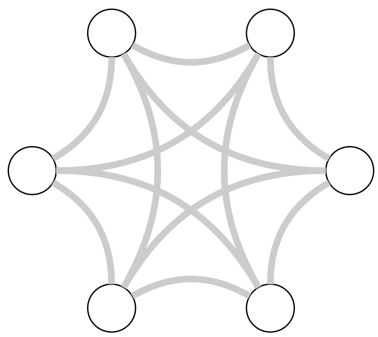
4. Construct a graph with vertices of degree 4, 4, 4, 4, 4 or explain why no such graph exists.

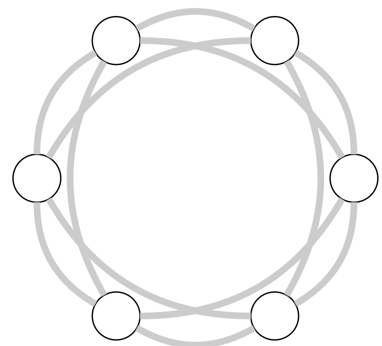
5. Construct a graph with vertices of degree 1, 2, 3, 2 or explain why no such graph exists.

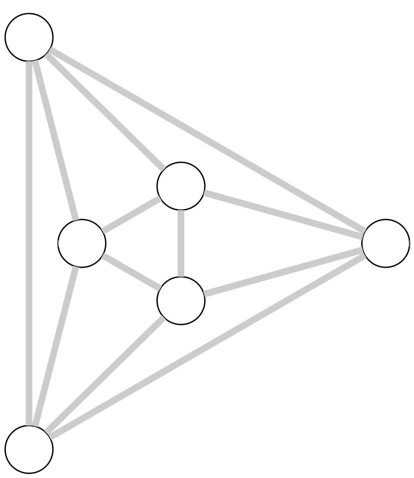
Part IV: Finding the Euler circuits

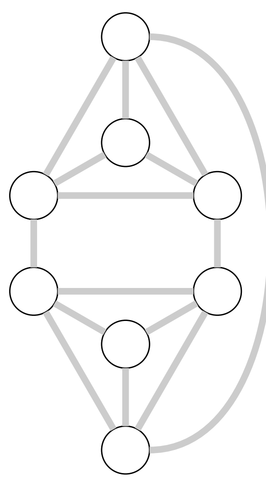
1. For each graph label the edges $1, 2, 3, \ldots$ in order of an Euler circuit or Euler path.

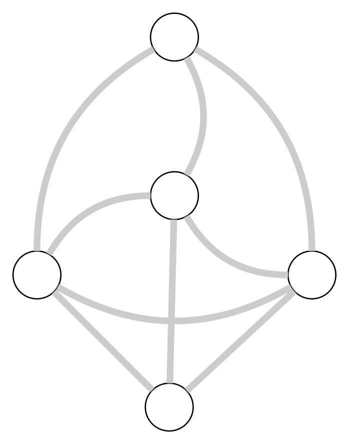












Part V: No, really, find them!

Label the degrees of each vertex, and then find optimal Eulerizations. Each of the graphs is connected.